A Gentle Introduction to DSGE Models

From Model Building to Welfare Evaluation with Dynare

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Roadmap

One economy, built before the break and evaluated after it.

PART I

- Why DSGE models?
- The anatomy of a model
- The running example, in Dynare
- Hands-on: impulse responses

PART II after a 15-minute break

- Approximation order and risk
- Three welfare measures
- Welfare in Dynare
- Hands-on: welfare evaluation

Part II adds exactly one equation to the model built in Part I.

01 Why DSGE Models?

What is a DSGE model?

A small artificial economy — a laboratory for policy.

D Dynamic	today's choices hinge on expectations of tomorrow
S Stochastic	random shocks drive the fluctuations
G General	all agents and markets in one system
E Equilibrium	prices make plans mutually consistent

Change a rule, re-run the economy, compare outcomes.

Why policy institutions build them

Because counterfactuals need structure, not correlations.

- **Regime changes** — expectations shift with policy (Lucas, 1976)
- **Internal consistency** — one set of preferences explains every result
- **Welfare** — household utility is a built-in yardstick

Central banks and international institutions run them as part of a suite.

→ who runs what

What DSGE models are not

Knowing the limits is part of using the tool.

- Not the best **forecasters** — statistical models often win short horizons
- Not **sectorally detailed** — one or a few goods, by construction
- Not **literal** — deliberately stylized; the value is transparent reasoning

So the question is never “DSGE or not?” but “which tool for this question?”

Where DSGE sits in the policy toolbox

Different questions, different canonical forms. Only DSGE carries $\mathbb{E}_t$.

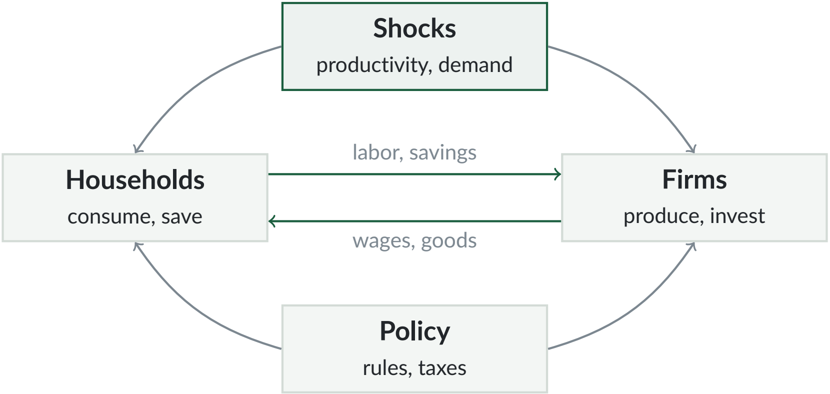
Tool	Canonical form	Niche
Macroeconometric	$y_{i,t} = \beta_i' x_{i,t} + u_{i,t}$	forecasting, scenarios
CGE	$F(p, q; \tau) = 0$	sectoral and tax incidence
VAR	$y_t = A(L)\, y_{t-1} + u_t$	responses in the data
Local projections	$y_{t+h} = \beta_h \varepsilon_t + \gamma_h' z_t$	responses, horizon by horizon
Microsimulation	$x_i^{\text{post}} = T(x_i^{\text{pre}})$	distributional detail
DSGE, one agent	$\mathbb{E}_t\, f(y_{t+1}, y_t, x_{t+1}, x_t) = 0$	**risk and welfare**
DSGE, many agents	$+\ \ \mu_{t+1} = \Gamma(\mu_t, \text{prices})$	**who gains, who loses**

The first four estimate equations; DSGE derives them jointly. Heterogeneity keeps that logic and adds the distribution μ_t as a state.

$\rightarrow$ DSGE vs. CGE

02 The Anatomy of a DSGE Model

Four blocks



Equilibrium = optimal decisions + market clearing + model-consistent expectations

Households and firms

One equation makes the model forward-looking.

$$u'(C_t) = \beta\, \mathbb{E}_t \left[u'(C_{t+1})\,(1 + r_{t+1}) \right]$$

- Consume now or save — the trade-off runs through **expectations**

$$Y_t = A_t\, K_{t-1}^{\alpha} H_t^{1-\alpha} \qquad r_t^k = \alpha\, Y_t / K_{t-1} \qquad w_t = (1-\alpha)\, Y_t / H_t$$

- Competition and constant returns $\Rightarrow$ factors paid marginal products

Shocks: persistence has an exact meaning

ρ sets how long. η sets how large.

$$a_t \equiv \ln A_t \iff A_t = e^{a_t}, \qquad a_t = \rho\, a_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \eta^2), \quad \sigma_a^2 = \frac{\eta^2}{1-\rho^2}$$

- The AR(1) is in **logs**, so $A_t > 0$ always and η reads as a percent

ρ	Half-life	90% decay
0.50	1.0 quarter	3.3 quarters
0.80	3.1 quarters	10.3 quarters
0.95	13.5 quarters $\approx$ 3.4 years	44.9 quarters $\approx$ 11 years

Note $\mathbb{E}[a] = 0$ but $\mathbb{E}[A] = e^{\sigma_a^2/2} = 1.0005 \neq 1$ — harmless now, decisive after the break.

Policy enters as a rule

Not a one-off intervention — a rule agents anticipate.

$$R_t = \overline{R} \left(\frac{\pi_t}{\overline{\pi}} \right)^{\rho_\pi} \left(\frac{Y_t}{\overline{Y}} \right)^{\rho_y} e^{\epsilon_t^R}$$

- Change (ρ_π, ρ_y) and private behavior changes with it
- That is exactly what makes rules **rankable by welfare** after the break
- The benchmark model is a chassis: housing, credit, open economy, fiscal

$\rightarrow$ a fiscal block bolted onto today's model

Equilibrium and solution

Stack everything into one system; solve for decision rules.

$$\mathbb{E}_t\, f(y_{t+1}, y_t, x_{t+1}, x_t) = 0$$

- x_t **states** — inherited: capital, shock levels
- y_t **controls** — chosen: consumption, and later welfare itself
- Solving $=$ finding the map from states to choices; Dynare automates it

How that map is approximated decides whether welfare analysis is possible at all.

solution details $\rightarrow$

03 The Running Example, in Dynare

The running example

Neoclassical growth, fixed labor. Used in both halves of the lecture.

$$
\begin{aligned}
\max_{C_t, K_{t+1}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} \\
\text{s.t.} \quad & C_t + K_{t+1} = A_t K_t^{\alpha} + (1-\delta) K_t \\
& \ln A_{t+1} = \rho \ln A_t + \eta\, \epsilon_{t+1}
\end{aligned}
$$

Minimal but complete, with an analytical steady state — and nothing gets swapped out over the break.

Equilibrium conditions

Three equations. That is the entire model.

$$
\begin{aligned}
C_t^{-\gamma} &= \beta\, \mathbb{E}_t\left[C_{t+1}^{-\gamma}\left(\alpha A_{t+1} K_{t+1}^{\alpha-1} + 1 - \delta\right)\right] \\
C_t + K_{t+1} &= A_t K_t^{\alpha} + (1-\delta) K_t \\
\ln A_{t+1} &= \rho \ln A_t + \eta\, \epsilon_{t+1}
\end{aligned}
$$

- Give up one unit today; collect the marginal product tomorrow
- After the break, **one more equation** joins this list

The example, read as D–S–G–E

Every DSGE model has this anatomy. More blocks, same logic.

D Dynamic	capital links periods; the Euler equation carries $\mathbb{E}_t$
S Stochastic	A_t is hit every period; ρ and η shape the hit
G General	consumption, saving, production decided jointly
E Equilibrium	goods market clears; prices equal marginal products

The same model in canonical form

Two states, one control, one forward-looking equation.

$$y_t = C_t, \qquad x_t^1 = K_t, \qquad x_t^2 = \ln A_t$$

$$\mathbb{E}_t \begin{bmatrix} y_{1,t}^{-\gamma} - \beta y_{1,t+1}^{-\gamma} \left(\alpha \exp(x_{2,t+1})\, x_{1,t+1}^{\alpha-1} + 1 - \delta \right) \\ y_{1,t} + x_{1,t+1} - \exp(x_{2,t})\, x_{1,t}^{\alpha} - (1-\delta) x_{1,t} \\ x_{2,t+1} - \rho x_{2,t} \end{bmatrix} = 0$$

This is the object Dynare receives — and the object that gets approximated after the break.

Steady state

Patience sets the price of capital; technology sets the quantity.

$$1 = \beta\left(\alpha \bar{K}^{\alpha-1} + 1 - \delta\right) \quad \Longrightarrow \quad \bar{K} = \left(\frac{\alpha}{1/\beta - (1-\delta)}\right)^{\frac{1}{1-\alpha}}$$

$$\bar{Y} = \bar{K}^{\alpha}, \qquad \bar{I} = \delta \bar{K}, \qquad \bar{C} = \bar{K}^{\alpha} - \delta \bar{K}$$

- Required return $1/\beta - 1 + \delta = 0.035$ per quarter
- Analytical, so Dynare needs no guess — and typos surface immediately

Calibration

PARAMETERS, QUARTERLY

γ	2	risk aversion
α	1/3	capital share
β	0.99	$\approx 4\%$ annual real rate
δ	0.025	10% annual depreciation
ρ	0.95	TFP persistence
η	0.01	TFP innovation s.d.

IMPLIED STEADY STATE

$\bar{K}$	29.264
$\bar{Y}$	3.082
$\bar{I}$	0.732
$\bar{C}$	**2.350**
$\bar{I}/\bar{Y}$	23.7%

Remember $\bar{C} = 2.350$. After the break it becomes $\overline{V} = u(\bar{C})/(1-\beta) = -42.55$.

`part1_model.mod` — the model block

```
var k c a y i;               // y and i are definitions, for reporting
varexo e;
parameters gamma alpha beta delta rho eta;

model;
c^(-gamma) = beta*c(+1)^(-gamma)*(alpha*exp(a(+1))*k^(alpha-1)+(1-delta));
c + k = exp(a)*k(-1)^alpha + (1-delta)*k(-1);
a = rho*a(-1) + e;
end;
```

- **Timing**: `k` is chosen now, `k(-1)` is inherited
- Most Blanchard–Kahn failures are timing typos, not numerical trouble

`part1_model.mod` — solving it

```
steady_state_model;           // the analytical formulas, typed in
k = (alpha/(1/beta-(1-delta)))^(1/(1-alpha));
c = k^alpha - delta*k;
a = 0;   y = k^alpha;   i = delta*k;
end;

steady;   check;              // steady state, then Blanchard-Kahn

shocks;
var e = eta^2;                // the VARIANCE, not the s.d.
end;

stoch_simul(order=1, irf=40) y c i k a;
```

`order=1` is a deliberate choice — and we will see exactly what it costs. → options & errors

What to look for in the output

The model reproduces the business-cycle ranking unprompted.

	mean	**σ/mean**	**corr. with y**
output y	3.082	4.51%	1.00
consumption c	2.350	3.25%	0.95
investment i	0.732	9.70%	0.94

- Investment volatile, consumption smooth — **not** a calibration target
- `check`: 2 unstable eigenvalues for **2 forward-looking variables** — `c` and `a`, the two dated $t+1$ in the Euler equation
- One unstable root per jumper: fewer $\Rightarrow$ indeterminacy, more $\Rightarrow$ no stable path

Results live in `oo_`; the model lives in `M_`.

04 Hands-On: Impulse Responses

Running the model

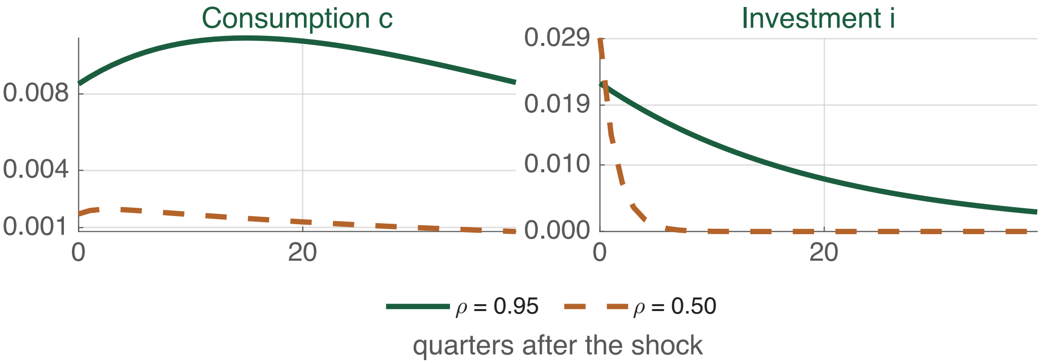
```
cd '~/Desktop/DSGE lectures/code files'
addpath('/Applications/Dynare/7.0-arm64/matlab')   % adjust to your setup

dynare part1_model        % the baseline
part1_compare_irfs        % the experiments
```

- The driver calls Dynare **once**, then re-solves with `set_param_value` + `stoch_simul`
- A second `dynare` call would clear the workspace — the classic trap
- Two experiments: **persistence** (ρ) and **volatility** (η)

→ installing Dynare

Experiment A — persistence



Output's impact is the same either way (0.031). What changes is the **split**: a lasting gain is consumed (c 0.0085), a fleeting one saved (i 0.0291).

Part I takeaways

- A DSGE model is optimizing agents $+$ shocks $+$ equilibrium, stacked into $\mathbb{E}_t f(\cdot) = 0$
- The Dynare path is short: equations $\rightarrow$ `.mod` $\rightarrow$ `steady` $\rightarrow$ `check` $\rightarrow$ `stoch_simul`
- Ranking policies means comparing economies that differ in **risk**

And a first-order solution is blind to risk. That is where we resume.

05 Approximation Order and Risk

The question after the break

Which policy is better — and by how much?

- The model carries its own yardstick: the **utility** that already drives every decision in it
- No welfare criterion has to be imported from outside
- What it adds to the toolbox: intertemporal, utility-based, **under uncertainty**

Computing it correctly has more traps than one would expect.

Welfare is just one more variable

Lifetime welfare satisfies a recursion, so Dynare can solve for it.

$$V_t = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j u(C_{t+j}) = u(C_t) + \beta\, \mathbb{E}_t V_{t+1}$$

`part1_model.mod` + one line = `part2_model.mod`

- V_t becomes an extra control: $y_t = [C_t, V_t]'$
- Steady state: $\overline{V} = u(\bar{C})/(1-\beta) = (-1/2.350)/0.01 =$ **-42.55**

The hard part is not *defining* welfare. It is *approximating* it.

Perturbation

Substitute the unknown rules into the model, then differentiate.

$$y_t = g(x_t, \sigma), \qquad x_{t+1} = h(x_t, \sigma) + \eta\sigma\epsilon_{t+1}$$

Put these back into $\mathbb{E}_t f(\cdot) = 0$. It must hold for *every* (x, σ), so it is an identity:

$$F(x,\sigma) \equiv \mathbb{E}_t\, f\big(g(h+\eta\sigma\epsilon', \sigma),\ g(x,\sigma),\ h+\eta\sigma\epsilon',\ x\big) = 0$$

- f is the **model** — known. g and h are the **unknowns**
- Differentiate F at $(\bar{x}, 0)$ and read off g's and h's derivatives, order by order (Schmitt-Grohé and Uribe, 2004)
- σ is a separate argument — that is what lets risk enter at all

`stoch_simul(order=k)` automates this.

$\rightarrow$ order-2 formulas

First order: certainty equivalence

$g_\sigma = h_\sigma = 0$ — the rules ignore the shock variance.

- Agents behave as if the future were certain
- You ran it before the break: doubling η rescaled the IRF, nothing more
- So **two policies with the same steady state tie exactly**

The comparison is empty by construction, not by coincidence. Welfare needs order two.

What changes at second order

Risk finally has a price.

- **Precautionary behavior** — agents respond to risk itself
- **Jensen's inequality** — $\mathbb{E}[u(C)] \neq u(\mathbb{E}[C])$: volatility is costly
- **Risk-corrected means** — the stochastic mean leaves the steady state
- **Rankings become possible** — same steady state, different risk

Order 2 is the workhorse; order 3 matters mainly for asset pricing. $\rightarrow$ pruning

06 Three Welfare Measures

Three welfare measures

They differ only in what they condition on.

$\overline{V}$ **Non-stochastic** – shocks off forever, $u(\overline{C})/(1-\beta)$

$\mathbb{E}V$ **Unconditional** – ergodic average for someone who has always lived there

$\mathbb{E}_{ss}V$ **Conditional** – start at the steady state, risk on from tomorrow

All three coincide at order 1. Their gaps *are* the risk effects.

$\rightarrow$ a fourth resting point

Consumption equivalents

Raw V is in utils. Report λ instead.

$$V^{\text{old}} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u((1-\lambda)\, C_t^{\text{new}}) \qquad \Longrightarrow \qquad \lambda = 1 - \left(\frac{V^{\text{old}}}{V^{\text{new}}}\right)^{\frac{1}{1-\gamma}}$$

- The fraction of consumption an agent would give up, every period
- Necessary because raw V shifts with β and γ for reasons that have nothing to do with welfare

Scale: Lucas (1987, 2003) puts the cost of business cycles near 0.05% of consumption. $\rightarrow$ derivations

Which one to report?

Question	Measure	Why
Cost of uncertainty itself	$\overline{V}$	the no-shock benchmark
Comparing long-run regimes	$\mathbb{E}V$	agents have lived under the policy
A one-time reform	$\mathbb{E}_{ss}V$	the transition matters
Communicating any of them	λ	utils mean nothing to anyone

Match the measure to the question, then report robustness across measures.

Ranking policy rules: three routes

From transparent to optimal.

- **Grid search** — loop over coefficients, re-solve at order 2; `part2_welfare_loop.m` is this template
- `osr` — Dynare picks coefficients of a *given* rule to minimize a quadratic loss → details
- **Ramsey** — the optimal-commitment benchmark → details

A variance-based loss is *not* automatically utility-based welfare.

07 Welfare in Dynare

`part2_model.mod` — one line longer

```
var k c a V;
varexo e;

model;
c^(-gamma) = beta*c(+1)^(-gamma)*(alpha*exp(a(+1))*k^(alpha-1)+(1-delta));
c+k = exp(a)*k(-1)^alpha+(1-delta)*k(-1);
a = rho*a(-1)+e;
V = c^(1-gamma)/(1-gamma)+beta*V(+1);      // <-- the only addition
end;
```

- The first three equations are character for character the Part I model
- V stays in **levels**: $\gamma > 1$ makes $V < 0$, so logs are undefined

`part2_model.mod` — steady state and solution

```
steady_state_model;
k = (alpha/(1/beta-(1-delta)))^(1/(1-alpha));
c = k^alpha-delta*k;
a = 0;
V = (c^(1-gamma)/(1-gamma))/(1-beta);      // Vbar = u(cbar)/(1-beta)
end;

stoch_simul(order=1, irf=120);
```

- $\bar{c} = 2.350 \;\Rightarrow\; \overline{V} = -42.553$, as promised before the break
- Four files in all: `part2_model.mod`, `part2_unadjusted.mod`, `part2_adjusted.mod`, `part2_welfare_loop.m`

Where welfare lives in the output

Three measures, three one-liners.

```
welfare_index = strmatch('V',M_.endo_names,'exact');

Vbar    = oo_.dr.ys(welfare_index);
EV      = oo_.mean(welfare_index);
EssV    = oo_.dr.ys(welfare_index) ...
          + 0.5*oo_.dr.ghs2(oo_.dr.inv_order_var(welfare_index));
```

- `ghs2` is the risk correction $g_{\sigma\sigma}\sigma^2$
- Perturbation-based — **no simulation involved**

→ where that formula comes from

Two ways to write one shock

UNADJUSTED — AR(1) IN LOGS

```
a = rho*a(-1)+e;
A = exp(a);
```

$$\mathbb{E}[A] \approx 1 + \frac{1}{2}\frac{\eta^2}{1-\rho^2} > 1$$

ADJUSTED — AR(1) IN LEVELS

```
A = (1-rho)+rho*A(-1)+e;
```

$\mathbb{E}[A] = 1$ exactly, for any η

- In logs, raising η also raises **average** productivity — Jensen
- Identical at first order; at second order they answer **different questions**

Why the specification matters

It is an experimental-design problem, not a coding problem.

- Asking “how costly is volatility?” means raising η and nothing else
- **Unadjusted**: risk arrives bundled with free extra productivity
- **Adjusted**: average TFP pinned at 1, so the risk cost is isolated

Neither is wrong as a model. Only one answers the question. When a parameter moves, check what else moved.

`part2_welfare_loop.m` — the welfare loop

```
vector_eta = 0.01:0.01:0.05;

for eta_i = 1:number_eta
    eta = vector_eta(eta_i);
    dynare part2_unadjusted.mod noclearall nolog   % or part2_adjusted.mod
    ...store the three welfare measures...
end
```

- `noclearall` is essential — otherwise the loop variable vanishes
- Replace η with policy-rule coefficients and this becomes a policy evaluation tool

08 Hands-On: Welfare Evaluation

Baseline, then order 2

```
dynare part2_model                    % find V in the output: Vbar = -42.55
                                      % at order=1, mean(V) = steady state of V

stoch_simul(order=2, irf=120);        % change this line, rerun
```

- At order 1 the mean of V **equals** its steady state — certainty equivalence, live
- At order 2 the means of V, c, k separate from the steady state

This one-line change is the entry ticket to welfare analysis.

The welfare loop, in three stages

Three runs. The differences between them are the whole lesson.

1. Run `part2_welfare_loop.m` as distributed — unadjusted, `order=1`
2. Set `order=2` in `part2_unadjusted.mod`, run again
3. Set `ind_unadjusted_a = false` in `part2_welfare_loop.m` — the adjusted file, already at order 2

Expected results on the next two slides.

→ troubleshooting

Stage 1 — first order

Every row flat. Volatility does nothing.

Unadjusted, order 1	$\eta = 0.01$	0.02	0.03	0.04	0.05
$\mathbb{E}(A)$	1.000	1.000	1.000	1.000	1.000
$\mathbb{E}(V)$	−42.553	−42.553	−42.553	−42.553	−42.553
$\mathbb{E}_{ss}(V)$	−42.553	−42.553	−42.553	−42.553	−42.553

A policy comparison run at order 1 reports a perfect tie — a numerical artifact, not a finding.

Stages 2 and 3 — second order

Unadjusted, order 2	$\eta = 0.01$	0.02	0.03	0.04	0.05
$\mathbb{E}(A)$	1.0005	1.0021	1.0046	1.0082	1.0128
$\mathbb{E}(V)$	−42.552	−42.549	−42.545	−42.538	−42.530
$\mathbb{E}_{ss}(V)$	−42.558	−42.574	−42.600	−42.636	−42.682

Adjusted, order 2	$\eta = 0.01$	0.02	0.03	0.04	0.05
$\mathbb{E}(A)$	1.0000	1.0000	1.0000	1.0000	1.0000
$\mathbb{E}(V)$	−42.585	−42.680	−42.839	−43.062	−43.348
$\mathbb{E}_{ss}(V)$	−42.584	−42.677	−42.833	−43.051	−43.331

$\overline{V} = -42.553$ in both panels, throughout.

Reading the tables

Same model, same code — opposite conclusions.

- **Unadjusted**: $\mathbb{E}V$ *rises* with volatility while $\mathbb{E}_{ss}V$ *falls*
- Nothing is miscomputed — the level benefit and the risk cost are entangled
- **Adjusted**: $\mathbb{E}[A]$ stays at 1, both measures fall monotonically — a clean risk cost

In consumption equivalents, the adjusted gap between $\eta = 0.01$ and 0.05 is $\lambda \approx 1.7\%$ per period.
One line in the shock process decided the sign of the answer.

Checklist

Six things that go wrong in practice.

1. **Order** — at least 2 for anything involving risk
2. **Shock specification** — logs or levels, chosen deliberately
3. **Conditioning** — know which steady state your measure sits at → options
4. **Perturbation vs. simulation** — simulated means need pruning, burn-in, and patience
5. **Units** — report λ; differences only, same β and utility
6. **Accuracy** — all measures must coincide at order 1; spot-check order 3

Takeaways

One economy, three equations, plus a welfare recursion.

- Welfare needs **order two** — at first order every same-steady-state policy ties
- Know the three measures, match them to the question, report λ
- The **shock specification** is a modeling decision with welfare consequences
- `part2_welfare_loop.m` + a `.mod` file is a reusable policy-evaluation pipeline

$\rightarrow$ policy applications: monetary, macroprudential, property tax

References — solution method

- Schmitt-Grohé, S. and M. Uribe (2004) — "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function," *JEDC* 28(4), 755–775 (· Matlab code)
- Schmitt-Grohé, S. and M. Uribe (2007) — "Optimal Simple and Implementable Monetary and Fiscal Rules," *JME* 54(6), 1702–1725
- Kim, J., S. Kim, E. Schaumburg, and C. Sims (2008) — "Calculating and Using Second-Order Accurate Solutions of Discrete Time Dynamic Equilibrium Models," *JEDC* 32(11), 3397–3414
- Andreasen, M., J. Fernández-Villaverde, and J. Rubio-Ramírez (2018) — "The Pruned State-Space System for Non-Linear DSGE Models," *ReStud* 85(1), 1–49

References — welfare, background, software

- Lucas, R. (2003) — "Macroeconomic Priorities," *AER* 93(1), 1–14; the earlier calculation is in *Models of Business Cycles* (1987)
- Coeurdacier, N., H. Rey, and P. Winant (2011) — "The Risky Steady State," *AER P&P* 101(3), 398–401
- Lucas, R. (1976) — "Econometric Policy Evaluation: A Critique," *Carnegie-Rochester Conf. Series on Public Policy* 1, 19–46
- Galí, J. (2015) — *Monetary Policy, Inflation, and the Business Cycle*, 2nd ed., Princeton

- Dynare Reference Manual · Collard (2001), *Stochastic Simulations with Dynare*
- Eric Sims' lecture notes · Johannes Pfeifer's `DSGE_mod` replication library

Appendix

DSGE models at policy institutions

Institution	Model	Documentation
Federal Reserve Board	SIGMA	Erceg, Guerrieri, and Gust (2006), *IJCB* 2(1)
European Central Bank	NAWM	Christoffel, Coenen, and Warne (2008), ECB WP 944
Bank of Canada	ToTEM	Murchison and Rennison (2006), BoC TR 97
Sveriges Riksbank	Ramses II	Adolfson et al. (2013), Riksbank Occ. Paper 12
Bank of England	COMPASS	Burgess et al. (2013), BoE WP 471
IMF	GIMF	Kumhof et al. (2010), IMF WP 10/34

On the macroeconometric side: Brayton, Laubach, and Reifschneider (2014), "The FRB/US Model," FEDS Notes.

← back

DSGE and CGE: complements, not substitutes

Same equilibrium logic. Different question.

CGE

$$F(p,q;\tau) = 0$$

- counterfactual $=$ re-solve at τ'
- rich sectoral and tax detail
- usually no $\mathbb{E}_t$, no shock distribution

DSGE

$$\mathbb{E}_t\, f(\cdot) = 0$$

- counterfactual $=$ change a *rule*
- expectations and dynamics
- welfare under uncertainty

Sectoral incidence $\rightarrow$ CGE. Expectations and risk $\rightarrow$ DSGE.

$\leftarrow$ back

Solution method and Blanchard–Kahn

Unique stable solution: as many unstable eigenvalues as jumpers.

$$y_t = g(x_t, \sigma), \qquad x_{t+1} = h(x_t, \sigma) + \eta \sigma \epsilon_{t+1}$$

- Perturbation expands the equilibrium system around the steady state
- `check` verifies the Blanchard–Kahn rank and order conditions
- In the running example: 2 unstable eigenvalues, 2 forward-looking variables

← back

A fiscal block on the same model

Decentralize and add a government: three equations become nine.

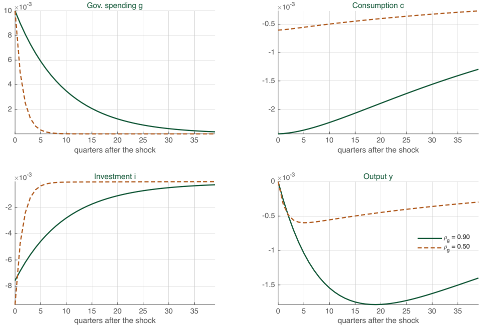
$$
\begin{aligned}
\text{households} \quad & c_t^{-\gamma} = \beta \mathbb{E}_t\left[c_{t+1}^{-\gamma}(r_{t+1}^k + 1 - \delta)\right] \\
\text{firms} \quad & y_t = e^{a_t} k_{t-1}^{\alpha}, \quad r_t^k = \alpha y_t / k_{t-1}, \quad w_t = (1-\alpha) y_t \\
\text{resources} \quad & c_t + i_t + g_t = y_t, \quad k_t = i_t + (1-\delta) k_{t-1} \\
\text{policy} \quad & t_t = g_t, \quad g_t = (1-\rho_g)\bar{g} + \rho_g g_{t-1} + \varepsilon_{g,t}
\end{aligned}
$$

Lump-sum taxes and fixed labor $\Rightarrow \bar{k}, \bar{y}, \bar{\imath}$ unchanged; $\bar{c}$ absorbs $\bar{g}$ one-for-one (1.734 at $\bar{g} = 0.2\bar{y}$).

`code files/extensions/session1_model.mod` $\rightarrow$ IRFs

$\leftarrow$ back

Fiscal extension — spending-shock IRFs



Crowding out: on impact c falls 0.0024 and i falls 0.0076 — together the 0.01 shock — while y cannot move. With fixed labor and flexible prices there is no demand channel, hence no positive multiplier.

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Official Dynare examples

Every installation ships teaching models.

- Collard (2001), RBC with *endogenous labor* and two correlated shocks
  - Dynare $\leq$ 5: `examples/example1.mod`
  - Dynare 7: `examples/stochastic_simulations/collard_2001_theoretical_moments.mod`
- `nk_baseline.mod` — the New Keynesian model (Born and Pfeifer); the full list is in the manual
- Adding sticky prices gives the textbook three-equation form (Galí, 2015); policy experiments then vary ϕ_π, ϕ_y and rank the rules by welfare
- Johannes Pfeifer's `DSGE_mod` — a large curated replication library

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`stoch_simul` options and common errors

- `order=1|2|3` (default **2**); `irf=N`; `periods=N`; `hp_filter=1600`; `nograph`; `noprint`; `pruning`
- "steady state could not be found" — bad guess or mistyped equation (`resid;` helps)
- "Blanchard & Kahn conditions are not verified" — usually a timing error (`k` vs. `k(-1)`)
- "Undeclared symbol" — missing from `var` / `varexo` / `parameters`
- `dynare` undefined — the Dynare `matlab` folder is not on the path

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Installing Dynare

```
addpath('/Applications/Dynare/7.0-arm64/matlab')
dynare_version
```

- Free and GPL, from dynare.org/download
- macOS: `/Applications/Dynare/<version>/`; Windows: `c:\dynare\<version>\`
- Requires MATLAB R2018b or later for Dynare 7, or GNU Octave

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The second-order decision rule

Uncertainty enters through a single constant.

$$y_t \approx \bar{y} + \tfrac{1}{2} g_{\sigma\sigma}\sigma^2 + g_x \hat{x}_t + \tfrac{1}{2} g_{xx}(\hat{x}_t \otimes \hat{x}_t) + \dots$$

- $\hat{x}_t = x_t - \bar{x}$; the cross terms in σ vanish ($g_{\sigma x} = g_{\sigma u} = 0$)
- Dynare's names: $\mathtt{ghx}$, $\mathtt{ghu}$, $\mathtt{ghxx}$, $\mathtt{ghuu}$, $\mathtt{ghxu}$, and $\mathtt{ghs2} = g_{\sigma\sigma}\sigma^2$
- At $\hat{x}_t = 0$: $\mathbb{E}_{ss} V = \overline{V} + \tfrac{1}{2}\mathtt{ghs2}_V$
- $\mathbb{E}V$ also reflects the ergodic distribution of $\hat{x}_t$ – hence $\mathbb{E}V \neq \mathbb{E}_{ss} V$

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Pruning

Simulating a quadratic rule can explode even when the model is stable.

```
stoch_simul(order=2, periods=10000, pruning);
```

- Squares of past states compound; pruning uses the first-order path inside the quadratic terms (Kim et al., 2008; Andreasen et al., 2018)
- Not needed in this lecture — `part2_welfare_loop.m` uses theoretical moments, so nothing is simulated

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Three resting points

Know which one your welfare measure sits at.

- **Deterministic steady state** — shocks off forever; what `steady` computes
- **Stochastic (risky) steady state** — no shocks realized, but risk expected (Coeurdacier, Rey, and Winant, 2011); iterate the rule with zero shocks
- **Ergodic mean** — the average over the stochastic distribution, `oo_.mean` at order 2

Conditional welfare is usually evaluated at the deterministic steady state, as in `part2_welfare_loop.m`. State which convention you use.

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Consumption equivalents — derivations

- CRRA, $\gamma \neq 1$: scaling C by $(1-\lambda)$ scales V by $(1-\lambda)^{1-\gamma}$

$$(1-\lambda)^{1-\gamma} V^{\text{new}} = V^{\text{old}} \;\Rightarrow\; \lambda = 1 - \left(\frac{V^{\text{old}}}{V^{\text{new}}}\right)^{\frac{1}{1-\gamma}}$$

- Log utility ($\gamma = 1$):

$$\lambda = 1 - \exp\Big((1-\beta)\left(V^{\text{old}} - V^{\text{new}}\right)\Big)$$

- Caveat: same β and same utility across regimes. With utility over consumption *and* leisure, scale only the consumption argument and re-derive

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Troubleshooting the hands-on

- "`eta is undefined`" — you ran the `.mod` file directly; run it via `part2_welfare_loop.m`, which uses `noclearall`
- All welfare rows identical — the file being run is still at `order=1`
- Results look stale — each `dynare` call overwrites `oo_`; store what you need first
- `steady_state_model` out of sync — edit the equations, edit the steady state too
- Explosive simulations at order ≥ 2 with `periods>0` — add `pruning`

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Optimal simple rules (`osr`)

```
osr_params rho_pi rho_y;
optim_weights;
  pi 1;       % weight on inflation variance
  y  0.5;     % weight on output variance
end;
osr;
```

- Minimizes a weighted sum of unconditional variances over the listed coefficients, using the **first-order** solution (Dynare 7 requires `order=1` in this mode)
- A quadratic loss is a stand-in for welfare unless derived from utility

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Ramsey policy

The optimal-commitment benchmark.

- Maximize a declared planner objective subject to *all* private-sector equilibrium conditions
- Workflow: `planner_objective` $\rightarrow$ `ramsey_model` $\rightarrow$ `stoch_simul` $\rightarrow$ `evaluate_planner_objective`
- The λ-gap between a simple rule and Ramsey is the **price of simplicity** — often small for well-chosen rules (SGU, 2007)

Not a black box: the answer depends on the objective, the instrument, and the commitment assumption.

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Policy applications

Same pipeline, different rule.

- **Monetary** — rank Taylor-rule coefficients $(\rho_R, \rho_\pi, \rho_y)$ by λ

$$R_t = R_{t-1}^{\rho_R}\left[\overline{R}(\pi_t/\overline{\pi})^{\rho_\pi}(Y_t/\overline{Y})^{\rho_y}\right]^{(1-\rho_R)} e^{\epsilon_t^R}$$

- **Macroprudential** — how tightly should an LTV cap respond to credit and house prices?
- **Property taxation** — welfare effects of τ including the transition, so conditional welfare is the natural measure

$$C_t + K_{t+1} + q_t H_{t+1} = A_t K_t^\alpha + (1-\delta)K_t + (1-\tau)q_t H_t$$

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