Welfare Analysis using Dynare

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References

- Schmitt-Grohé and Uribe's paper and lecture notes
- Eric Sims' lecture notes
- Dynare
- Johannes Pfeifer's collection of Dynare models

For many DSGE models

$$\mathbb{E}_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0 \tag{1}$$

- x_t: predetermined (state); y_t: non-predetermined (control)
- $x_t = [x_t^1; x_t^2]'$ where x_t^1 endogenous and x_t^2 exogenous
- $x_{t+1}^2 = \tilde{h}(x_t^2, \sigma) + \bar{\eta}\sigma\epsilon_{t+1}$

Example—Neoclassical model with fixed labor

- Consider

$$\max_{C_t, K_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} \tag{2}$$

$$\text{s.t. } C_t + K_{t+1} = A_t K_t^{\alpha} + (1-\delta) K_t \tag{3}$$

$$\ln A_{t+1} = \rho \ln A_t + \eta_A \epsilon_{t+1} \tag{4}$$

- FOC

$$C_t^{-\gamma} = \beta \mathbb{E}_t C_{t+1}^{-\gamma} \left(\alpha A_{t+1} K_{t+1}^{\alpha-1} + 1 - \delta \right) \tag{5}$$

(1)'s representation

- Let $y_t = C_t$, $x_t^1 = K_t$, $x_t^2 = \ln A_t$; so, $x_t = [K_t, \ln A_t]'$
- It follows that

$$\begin{aligned} &\mathbb{E}_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0 \\ &= \mathbb{E}_t \begin{bmatrix} y_{1,t}^{-\gamma} - \beta y_{1,t+1}^{-\gamma} \left(\alpha \cdot \exp(x_{2,t+1}) \cdot x_{1,t+1}^{\alpha-1} + 1 - \delta \right) \\ y_{1,t} + x_{1,t+1} - \exp(x_{2,t}) \cdot x_{1,t}^{\alpha} - (1-\delta) x_{1,t} \\ x_{2,t+1} - \rho x_{2,t} \end{bmatrix} \end{aligned} \tag{6}$$

Perturbation

- Key idea is to interpret the solution as a function of x_t and σ

$$y_t = g(x_t, \sigma) \tag{7}$$
$$x_{t+1} = h(x_t, \sigma) + \eta\sigma\epsilon_{t+1} \tag{8}$$

- A local approximation that is valid in the neighborhood of given $(\tilde{x}, \tilde{\sigma})$
- Focusing on non-stochastic steady state $(\bar{x}, 0)$
- Taylor expansion at of F, g, h at $(\bar{x}, 0)$ plus some linear algebra

First-, second-order, or beyond?

- First-order: the certainty equivalence principle holds ($g_\sigma = h_\sigma = 0$)
- Policy functions independently on $\Sigma_\epsilon \to$ No effect of uncertainty
  - Cannot study welfare! Why?
    Any policies with same steady state yield same welfare
- We need at least a second-order approximation to study welfare
- For higher-order, it might be helpful but needs pruning and is unclear

Welfare measure

- Welfare defined as the present discounted value of lifetime utility

$$V_t = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j u(C_{t+j}) = u(C_t) + \beta \mathbb{E}_t V_{t+1} \tag{9}$$

- Likewise, $y_t = [C_t, V_t]'$ so $V_t = g^V(x_t, \sigma)$
- Welfare cost λ (if CRRA utility with γ)

$$V^{old} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left((1-\lambda)C_t^{new}\right) \tag{10}$$

(Un)conditional welfare

- Non-stochastic welfare (no shocks): $\bar{V}$
  - Steady-state value without shocks, deterministic
- Unconditional welfare: $\mathbb{E}V$
  - Average welfare with shocks, stochastic mean of V
- Conditional welfare: $\mathbb{E}_t V$
  - Conditional on being in the steady state, but know that the model is stochastic so that shocks may happen in the future (transition)
  - Need to adjust the deterministic mean with $\frac{1}{2} g^V_{\sigma\sigma} \sigma^2$

Dynare implementation

- Non-stochastic welfare (no shocks): $\bar{V}$
  - `oo_.dr.ys(welfare_index)`
- Unconditional welfare: $\mathbb{E}V$
  - `oo_.mean(welfare_index)`
- Conditional welfare: $\mathbb{E}_{ss}V$
  - `oo_.dr.ys(Welfare_index)`
    `+ 0.5 * oo_.dr.ghs2(oo_.dr.inv_order_var(welfare_index))`

The example

- Recall equilibrium conditions

$$C_t^{-\gamma} = \beta \mathbb{E}_t C_{t+1}^{-\gamma} \left(\alpha A_{t+1} K_{t+1}^{\alpha-1} + 1 - \delta \right) \tag{11}$$

$$C_t + K_{t+1} = A_t K_t^{\alpha} + (1-\delta) K_t \tag{12}$$

$$\ln A_{t+1} = \rho \ln A_t + \eta_A \epsilon_{t+1} \tag{13}$$

- To study welfare, add

$$\boldsymbol{V_t = u(C_t) + \beta \mathbb{E}_t V_{t+1}} \tag{14}$$

Welfare results of varying η_A

- First-order: All welfare measures are the same
- How to cook in exogenous processes matters!
- Larger shock variance, larger difference between $\mathbb{E}V$ and $\mathbb{E}_{ss}V$

Applications

- Monetary policy (Taylor rule)

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R)\rho_\pi \hat{\pi}_t + (1 - \rho_R)\rho_y \hat{y}_t + \epsilon_t^R \tag{15}$$

- Macroprudential policy

$$\widehat{LTV}_t = \rho_B \hat{B} + \rho_q \hat{q} \tag{16}$$

- Housing transaction or property tax

Some tips

- Use `set_param_value` to loop over parameters
- Welfare weighs for two agents (like normalization)

$$W = (1-\beta_L)V^L + (1-\beta_H)V^H = u(C^L) + u(C^H) \tag{17}$$

- Use Dyanre to linearize Phillips curve to keep higher-order effects
  - Need to get a recursive representation
  - c.f. Johannes Pfeifer's `Gali_2015_chapter_3_nonlinear.mod`