# “Whatever it takes”: a good communication strategy?*

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**Abstract**

What is the optimal communication policy for a central bank facing an inattentive public? We develop a model of central bank communication in a Bayesian persuasion framework, incorporating information processing costs and heterogeneous attention budgets among individuals. The central bank seeks to stabilize the economy by shaping inflation expectations in response to fundamental shocks. When shocks are large, it is optimal to remain deliberately vague to leverage inflation surprises that mitigate unemployment fluctuations. When shocks are small, the central bank communicates informatively, balancing message precision against audience reach. We further examine how belief heterogeneity, adaptive expectations, the share of inattentive individuals, and skewed attention budget distributions shape the optimal communication strategy. Our findings underscore the strategic role of central bank communication as a flexible and effective instrument for macroeconomic stabilization.

**Keywords**: Central Bank Communication, Bayesian Persuasion, Rational Inattention, Inflation Expectations, Monetary Policy, Macroeconomic Stabilization

**JEL Classifications**: D83, E52, E58, E61

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# 1 Introduction

The primary objective of central banks is to maintain price stability. In practice, most countries adopt an inflation-targeting regime, wherein the central bank sets a specific inflation target and adjusts its policy instruments to keep inflation close to this target. Although central banks operate independently of political authorities, they may consider deviating from their inflation target temporarily in the face of extraordinary economic disruptions—such as those induced by exogenous shocks—to support broader macroeconomic objectives like output or employment stabilization. In such cases, the central bank may commit to introducing some flexibility into its targeting regime contingent on the shock's occurrence.

The effectiveness of monetary policy in influencing real outcomes hinges critically on how economic agents—or, more broadly, individuals—form their inflation expectations. Communication thus becomes a crucial policy tool for shaping these expectations (Casiraghi and Perez, 2022). A central bank not only supplies information but also strategically designs it to influence public beliefs. This communication may take various forms, ranging from technical reports to policy speeches. While technical reports are typically more precise, they may be less effective in reaching the public, as individuals often find them too complex or inaccessible. In contrast, speeches, despite being less detailed, tend to capture more attention.

Central banks often choose to communicate with intentionally vague terms, even during episodes of heightened economic stress. Such statements can be effective precisely because of their simplicity, especially when individuals face cognitive constraints in processing information. A prominent example is the speech delivered by then-ECB President Mario Draghi in 2012 at the peak of the Eurozone sovereign debt crisis:

> *"Within our mandate, the ECB is ready to do* ***whatever it takes*** *to preserve the euro. And believe me, it will be enough."*

Despite its lack of technical detail, the speech is widely credited with calming financial markets and restoring confidence in the euro.

This statement was intentionally vague: while it conveyed a strong commitment to preserving the monetary union and successfully reassured financial markets, it revealed little about the central bank's expectations regarding the evolution of the crisis or the specific instruments it would use. Why would it be optimal for a central bank to withhold such information in the face of a severe crisis? This paper offers a novel, theoretically grounded explanation for that communication strategy.

We study a partial equilibrium model of central bank communication featuring Bayesian persuasion under individual information processing costs and attention budgets. The economy can be in one of two states, *weak* or *strong*, each associated with a state-dependent fundamental shock that drives the economy away from its steady state. These shocks are interpreted as an unemployment shock in the weak state and an employment shock in the strong state. The central bank and individuals may hold different prior beliefs about the likelihood of each state. The central bank seeks to stabilize the economy by minimizing inflation and output gaps across states, captured by a quadratic loss function. We assume the central bank commits to a state-contingent monetary policy to stabilize the economy. According to the classical Phillips curve, the unemployment gap depends on the slope of the curve—that is, the sensitivity of unemployment to inflation surprises—and the inflation surprise itself, defined as the gap between actual inflation and individuals' expected inflation. Through information design, the central bank can influence the magnitude of this inflation surprise.

We consider a Sender–Receiver game in which the central bank (the sender) possesses commitment power—formally modeled as Bayesian persuasion following Kamenica and Gentzkow (2011)—while individuals (the receivers) face limited attention. The central bank can influence individuals' posterior beliefs about the underlying state and, consequently, their inflation expectations by designing a state-contingent communication strategy. However, consuming information is costly. Following Sims (2003), we model these information processing costs using entropy.[1] In essence, the cost increases with the distance between the posterior belief induced by the message and the individual's prior belief. That is, messages conveying more surprising or unfamiliar information are more difficult to process.

Each individual is endowed with an idiosyncratic attention budget, which determines whether they will engage with a message based on its complexity. Given these information processing costs and heterogeneous attention budgets, some individuals may ignore the central bank's message if its complexity exceeds their cognitive capacity. These inattentive individuals form inflation expectations based solely on their prior beliefs. In contrast, attentive individuals—those with

[1]Understanding a message requires cognitive effort, such as reading and interpreting content.

sufficiently high attention budgets—process the message and update their expectations based on the induced posterior beliefs.

Since individuals are rationally inattentive, the central bank faces a fundamental trade-off between message precision and message popularity when using communication to stabilize the economy. We are the first to examine this novel trade-off within a model of central bank communication under commitment.

As our benchmark, we consider a setting in which the central bank and individuals share identical, neutral prior beliefs, and the fundamental shocks are symmetric across states. The attention budget is uniformly distributed, and individuals form rational inflation expectations. Under this scenario, we theoretically characterize the optimal central bank communication strategy, which features two distinct regimes along the extensive margin: one where the bank communicates informatively, and another where it remains uninformative.

When the magnitude of the shocks exceeds the effective power of inflation surprises—defined as the product of the sensitivity parameter and the inflation surprise—the central bank optimally chooses to communicate uninformatively. By deliberately withholding information, it induces full-scale inflation surprises, which help offset the adverse effects of the shocks and thus stabilize the economy.

By contrast, when the effective power of inflation surprises exceeds the magnitude of the shocks, the central bank opts for informative communication to better align inflation expectations with actual inflation across states, thereby mitigating excessive inflation surprises and stabilizing the economy. However, the intensive margin—namely, the extent to which the central bank can communicate precisely—is constrained by the share of inattentive individuals. A higher share of inattentive agents reduces the effectiveness of complex messages, forcing the central bank to trade off precision for popularity. As a result, full revelation is never optimal unless attention constraints are entirely absent or the cost of processing information is negligible.

We then examine the role of belief heterogeneity in central bank communication. In practice, individuals and the central bank often hold different views regarding the likelihood of economic states—that is, they possess distinct prior beliefs. We show that non-neutral priors affect both the extensive and intensive margins of communication, though their impacts differ depending on whether the heterogeneity arises from individuals or the central bank.

When individual prior beliefs shift toward one particular state, that state becomes subjectively more plausible. As a result, inflation surprises diminish in the more plausible state but intensify in the less plausible one. The mismatch between actual and expected inflation in the implausible state grows as individuals assign greater probability to the plausible state. Once this mismatch becomes sufficiently large, the central bank optimally switches from an uninformative to an informative communication strategy, sending a more precise message about the implausible state to reduce its excessive inflation surprise.

This individual belief heterogeneity also influences the intensive margin. The central bank reduces message precision for the state perceived by individuals as more plausible, while increasing precision for the less plausible state to mitigate excessive inflation surprises. However, due to information processing costs, there exists a threshold in individual priors beyond which the central bank overturns this strategy: it finds it optimal to send a more precise message about the most plausible state in order to lower communication complexity and enhance message popularity.

The central bank's belief reflects its perceived priority regarding which state requires more urgent management of inflation surprises. As a result, the optimal communication strategy is to send a more precise message about the state the central bank considers more plausible. This improves the alignment between actual and expected inflation, thereby minimizing the expected loss in that state. However, in order to preserve message popularity among inattentive individuals, the central bank correspondingly reduces the precision of messages about the less plausible state.

There are more individuals with low attention budgets in practice. For example, many individuals in the United States lack financial literacy, as evidenced by their failure to correctly answer all of the Big Three financial literacy questions. To quantify this empirically relevant feature, we introduce the Kumaraswamy distribution to model the skewness of attention budgets toward lower values.[2] This framework enables us to examine how a population concentrated around low attention capacity affects optimal central bank communication. Intuitively, as the attention budget distribution becomes more right-skewed (i.e., with a higher share of low-budget individuals), the central bank optimally reduces message precision to lower complexity and reach a larger portion of the population.

Interestingly, we also uncover that the precision the central bank employs in

[2]The uniform distribution is a special case of the Kumaraswamy distribution. This desirable property allows for convenient comparisons across different distributional assumptions.

communication (intensive margin) is a non-trivial function of the overall share of inattentive individuals and the distribution of attention among them. Typically, the lower the average attention budget across individuals, the less precise the information the central bank is willing to provide to gain popularity. However, if the population includes a sufficiently large mass of fully attentive individuals and only a small share of inattentive individuals—most of whom have very limited attention budgets—the central bank may find it optimal to forgo communication with the inattentive group altogether. In such cases, it chooses to send fully informative messages targeted at the attentive individuals.

Rational expectations are often criticized as a strong and unrealistic assumption. To explore how alternative expectation formations affect central bank communication, we examine a setting in which individuals form irrational inflation expectations that are partially anchored to their prior beliefs. This adaptive expectation mechanism weakens the effectiveness of communication, as a message with a given level of precision becomes less effective at guiding irrational individuals toward posterior beliefs that support macroeconomic stabilization. As a result, compared to the rational expectations benchmark, the central bank optimally responds by communicating more precisely to correct the expectation misalignment, thereby better managing inflation surprises.

**Related Literature** Our paper contributes to two strands of literature: Bayesian persuasion and central bank communication.

First, we contribute to the literature on Bayesian persuasion, pioneered by Aumann, Maschler, and Stearns (1995) and Kamenica and Gentzkow (2011). We examine the problem of a sender—a central bank—who designs information for rationally inattentive (Sims, 2003) receivers (individuals) to influence their decision-making. Several studies have incorporated limited attention into Bayesian persuasion frameworks; see, for example, Bloedel and Segal (2020), Lipnowski, Mathevet, and Wei (2020), Lipnowski, Mathevet, and Wei (2022), Wei (2021), Matyskova and Montes (2023), and Innocenti (2024).

In our setting, the central bank and individuals have partially aligned objectives. The central bank can provide perfectly informative messages about the state of the economy, but doing so may exceed the cognitive capacity of inattentive individuals. In certain cases, the bank optimally chooses to remain uninformative when inflation surprises can be leveraged to offset real economic shocks. To our knowledge, we are the first to study information design with commitment in a central bank communication framework that explicitly accounts for rational inattention.

The literature on central bank communication has traditionally relied on cheap talk models, in which the central bank lacks commitment to its messaging strategy (Crawford and Sobel, 1982). Notable examples include Stein (1989), Moscarini (2007), and Bassetto (2019), where communication is inherently imperfect due to strategic interaction between the central bank and the public. By contrast, we assume full commitment to the communication strategy. In this setting, central bank communication emerges as a complementary instrument to inflation-targeting monetary policy, enabling the central bank to mitigate economic fluctuations without deviating from its policy rule. We contribute to this literature by providing the first theoretical characterization of the optimal communication strategy when the central bank faces inattentive individuals and aims to influence inflation expectations through Bayesian persuasion.

More recently, a growing body of research has applied Bayesian persuasion to analyze optimal central bank communication. For example, Herbert (2021) studies communication in a setting characterized by coordination externalities and belief heterogeneity, showing that countercyclical messaging emerges as optimal. The study most closely related to ours is Ko (2022), which corresponds to our benchmark framework when individuals are fully attentive.[3] In contrast to Ko (2022), our paper highlights the importance of audience characteristics in shaping the optimal communication strategy, focusing on rational inattention, belief heterogeneity, and departures from rational expectations. To the best of our knowledge, this is the first study to formally incorporate these dimensions into a unified Bayesian persuasion framework for central bank communication.

The rest of the paper is organized as follows. Section 2 outlines the model framework. In Section 3, we characterize the benchmark theoretical results for the optimal central bank communication strategy. Sections 4 to 6 examine how belief heterogeneity, shock asymmetry, the distribution of attention budgets, and irrational expectations shape the optimal communication design. Section 7 concludes with a discussion of potential extensions. Detailed mathematical derivations are provided in Appendix A, and additional robustness figures are included in Appendix B.

[3] In Ko (2022), the central bank receives a private, imperfect signal about the state of the economy and chooses the extent of disclosure. Our model yields qualitatively similar insights without assuming private information, thereby simplifying the informational structure while maintaining tractability.

## 2 Model

We study a partial equilibrium model of central bank communication, framed as a sender-receiver game between a central bank and individuals. Given a predetermined long-run monetary policy, the central bank chooses whether to send informative messages to influence the formation of individual inflation expectations, in order to stabilize the economy promptly in response to fundamental shocks. We examine the extent to which a central bank can leverage communication tools to lean against the wind in the short run—complementing its monetary policy rule without altering the pre-committed policy path and thereby avoiding any compromise of credibility. The central bank commits to a communication strategy prior to the realization of shocks, giving rise to an information design problem known as Bayesian persuasion.

The economy consists of two types of agents: a central bank and a unit-mass continuum of individuals. The central bank faces the following classical Phillips curve:

$$\left(u - u^N\right) = \omega - \gamma\left(\pi - \pi^e\right), \tag{1}$$

where $u$ denotes the unemployment rate, $u^N$ the natural rate of unemployment, $\omega$ an exogenous, state-dependent employment shock, $\gamma > 0$ the sensitivity of the unemployment gap to inflation surprises, $\pi$ actual inflation, and $\pi^e$ individual expected inflation. Hence, $\pi - \pi^e$ captures the inflation surprise, and $\gamma\left(\pi - \pi^e\right)$ reflects the extent to which unanticipated inflation can offset the adverse shock effect on the unemployment gap.

Equation (1) describes how unemployment responds to inflation surprises. When actual inflation exceeds expected inflation, unemployment falls below the natural rate. Conversely, when inflation is lower than expected, unemployment rises above it.

We assume that $\omega$ reflects a fundamental disturbance independent of inflation expectations and also serves as the state of the economy. It can take on two values: in the weak state, $\omega_1 > 0$, unemployment rises further above the natural rate; in the strong state, $\omega_2 < 0$, it falls below it. We define the state space as $\Omega := \{\omega_1, \omega_2\}$. To stabilize the economy in the face of unemployment fluctuations, the central bank can trade off higher inflation for lower unemployment, or vice versa.

The central bank manages actual inflation according to the following monetary policy rule:

$$\pi(\omega,\nu) := \begin{cases} \pi^T + \nu & \text{if } \omega = \omega_1, \\ \pi^T - \nu & \text{if } \omega = \omega_2, \end{cases} \tag{2}$$

where $\pi^T$ denotes the central bank's inflation target, and $\nu$ represents monetary flexibility.[4] According to equation (2), the central bank allows inflation to rise above target in the weak state to support economic recovery and lowers it below target in the strong state to guard against future inflationary pressures.

The ultimate goal of the central bank is to stabilize the economy by minimizing quadratic losses across states arising from unemployment and inflation gaps.[5] For a given monetary policy rule and a given state, the central bank's loss, denoted by $L(\omega,\nu)$, is given by:

$$L(\omega,\nu) := \left(u - u^N\right)^2 + \alpha\left(\pi - \pi^T\right)^2 \tag{3}$$

$$= [\omega - \gamma\left(\pi(\omega,\nu) - \pi^e\right)]^2 + \alpha\left(\pi(\omega,\nu) - \pi^T\right)^2, \tag{4}$$

where $\alpha$ denotes the relative weight the central bank places on the inflation gap relative to the unemployment gap. Equation (4) follows from substituting equations (1) and (2) into equation (3).

Individuals share a common prior belief $\mu_0$ that state $\omega_1$ will occur, while the central bank holds its own prior belief $\mu_0^c$. Accordingly, the beliefs assigned to state $\omega_2$ are $1 - \mu_0$ for individuals and $1 - \mu_0^c$ for the central bank. Individuals form their inflation expectations based on their beliefs, denoted by $\pi^e(\mu_0)$.

The central bank aims to minimize expected losses under its own prior, taking as given the inflation expectations formed by individuals. Its objective becomes:

$$\begin{aligned} &\mathbb{E}_{\mu_0^c} L(\mu_0,\nu) = \\ &\quad \mu_0^c \left\{ [\omega_1 - \gamma\left(\pi(\omega_1,\nu) - \pi^e(\mu_0)\right)]^2 + \alpha\left(\pi(\omega_1,\nu) - \pi^T\right)^2 \right\} + \\ &\quad (1-\mu_0^c)\left\{ [\omega_2 - \gamma\left(\pi(\omega_2,\nu) - \pi^e(\mu_0)\right)]^2 + \alpha\left(\pi(\omega_2,\nu) - \pi^T\right)^2 \right\}. \end{aligned} \tag{5}$$

While the central bank could adjust the flexibility of its monetary policy rule, doing so may come at the cost of reduced credibility—either by violating the predetermined rule or by inviting ex-post policy errors that under- or overshoot the

[4]For example, $\nu$ can be interpreted as the degree of money velocity or liquidity expansion the central bank uses to accommodate inflation.

[5]This parsimonious approach can be micro-founded. See, for example, Woodford (2003).

appropriate response to shocks. In the long run, it may be optimal to prudently re-formalize the monetary rule. In the short run, however, the central bank can instead rely on communication to influence individual inflation expectations as a means of stabilizing the economy.

This paper focuses on the latter case, in which the central bank communicates with individuals by providing information $\sigma$ to influence their posterior beliefs $\mu$ and, consequently, their inflation expectations. In what follows, we treat $\nu$ as exogenously given and, for notational simplicity, omit it as a function argument. Similarly, since the inflation gap is irrelevant for the information design, it is also omitted from the objective loss function. We therefore focus on how communication shapes the loss function $\mathbb{E}_{\mu_0^c} L(\mu, \sigma)$, given by:

$$\mu_0^c \left[\omega_1 - \gamma\left(\pi(\omega_1) - \pi^e(\mu)\right)\right]^2 + (1 - \mu_0^c)\left[\omega_2 - \gamma\left(\pi(\omega_2) - \pi^e(\mu)\right)\right]^2. \tag{6}$$

The information structure $\sigma$ consists of a set of messages $S := \{s_1, s_2\}$ and a family of conditional distributions $\{\sigma(\cdot \mid \omega)\}_{\omega \in \Omega}$ over $S$. That is, $\sigma : \Omega \to \Delta(S)$. Each message $s_j \in S$ sent by the central bank induces the following posterior belief $\mu_j$ about state $\omega_1$ among individuals:

$$\mu_j = \frac{\sigma(s_j \mid \omega_1)\mu_0}{\sigma(s_j \mid \omega_1)\mu_0 + \sigma(s_j \mid \omega_2)(1 - \mu_0)}. \tag{7}$$

As a result, $\sigma$ induces a distribution over posteriors $\mu_j$, that is, $\tau \in \Delta(\Delta(\Omega))$. The probability of each message $s_j$ (or, equivalently, each posterior $\mu_j$) as perceived by individuals is:

$$\tau_j = \sigma(s_j \mid \omega_1)\mu_0 + \sigma(s_j \mid \omega_2)(1 - \mu_0). \tag{8}$$

The martingale property—or Bayes plausibility condition—must hold: $\mathbb{E}_\tau[\mu_j] = \mu_0$. Without loss of generality, we assume that the optimal $\sigma$ satisfies $\mu_1 \geq \mu_0$ and $\mu_2 \leq \mu_0$.

Individuals can be either attentive or inattentive, with a share $\delta \in [0, 1]$ of the population being inattentive. Attentive individuals can process any information at no cost, whereas inattentive individuals are constrained by limited attention and can process only sufficiently simple information. Each inattentive individual $i$ is endowed with an attention budget $c_i$, drawn from an atomless distribution $F(\cdot)$ with support $[0, 1]$. An inattentive individual $i$ processes $\sigma$ if and only if

$c(\sigma) < c_i$, where $c(\sigma)$ denotes the cost of processing the information.

The cost of processing information is defined as:

$$c(\sigma) = \chi \left[ H(\mu_0) - \sum_{j \in \{1,2\}} \tau_j \cdot H(\mu_j) \right], \tag{9}$$

where $H(\cdot)$ denotes the Shannon entropy:

$$H(\mu) = -\left[\mu \ln(\mu) + (1-\mu)\ln(1-\mu)\right]. \tag{10}$$

When $\sigma$ is uninformative—i.e., $\mu_1 = \mu_2 = \mu_0$—the cost is zero: $c(\sigma) = 0$. When $\sigma$ is fully informative—i.e., $(\mu_1, \mu_2) = (1, 0)$—we have $H(\mu_1) = H(\mu_2) = 0$, and hence $c(\sigma) = \chi H(\mu_0)$. The scaling parameter $\chi$ is normalized as $\chi = [H(\mu_0)]^{-1}$, so that the cost of the most informative message spans the full support of the attention distribution, i.e., $F(\chi H(\mu_0)) = 1$.

It follows that the mass of individuals who pay attention to the central bank is $1 - \delta F(c(\sigma))$. The posterior beliefs of inattentive individuals remain at the common prior $\mu_0$. Therefore, the central bank's total expected loss under communication is given by:

$$W_\sigma := \delta F(c(\sigma))\mathbb{E}_{\mu_0^c} L(\mu_0, \sigma) + (1 - \delta F(c(\sigma))) \sum_{j=1}^{2} \tau_j^c \mathbb{E}_{\mu_j^c} L(\mu_j, \sigma), \tag{11}$$

where $\mu_j^c$ and $\tau_j^c$ denote, respectively, the posterior and the probability of message $s_j$ from the central bank's perspective, defined as:

$$\mu_j^c = \frac{\sigma(s_j \mid \omega_1)\mu_0^c}{\sigma(s_j \mid \omega_1)\mu_0^c + \sigma(s_j \mid \omega_2)(1-\mu_0^c)}, \tag{12}$$

$$\tau_j^c = \sigma(s_j \mid \omega_1)\mu_0^c + \sigma(s_j \mid \omega_2)(1-\mu_0^c). \tag{13}$$

Given (11), the central bank solves the following minimization problem:

$$
\begin{aligned}
\min_{\sigma} \quad & \delta F(c(\sigma)) \Big\{ \mu_0^c \left[\omega_1 - \gamma\left(\pi(\omega_1) - \pi^e(\mu_0)\right)\right]^2 \\
& + (1-\mu_0^c)\left[\omega_2 - \gamma\left(\pi(\omega_2) - \pi^e(\mu_0)\right)\right]^2 \Big\} \\
& + (1 - \delta F(c(\sigma))) \left\{ \sum_{j=1}^{2} \sigma(s_j \mid \omega_1)\mu_0^c \left[\omega_1 - \gamma\left(\pi(\omega_1) - \pi^e(\mu_j)\right)\right]^2 \right. \\
& \left. + \sum_{j=1}^{2} \sigma(s_j \mid \omega_2)(1-\mu_0^c) \left[\omega_2 - \gamma\left(\pi(\omega_2) - \pi^e(\mu_j)\right)\right]^2 \right\}.
\end{aligned}
\tag{14}
$$

## 3 How Should a Central Bank Communicate?

To better understand the trade-off in central bank communication between information precision and popularity, we consider a benchmark case under the following assumptions:

**Assumption 1.** *Prior beliefs are homogeneous and neutral:* $\mu_0 = \mu_0^c = \frac{1}{2}$.

**Assumption 2.** *Employment shocks are symmetric:* $\omega_1 = -\omega_2 = \omega$.

**Assumption 3.** *Individuals form rational inflation expectations:*

$$
\pi^e(\mu) = \mu\pi(\omega_1) + (1-\mu)\pi(\omega_2) = \pi^T - (1-2\mu)\nu. \tag{15}
$$

**Assumption 4.** *The attention budget distribution is uniform:* $F(c(\sigma)) = c(\sigma)$.

Under Assumptions 1–4, we can analytically characterize the optimal information structure that solves the minimization problem in Equation (14). In particular, the optimal communication policy becomes symmetric:

$$
\sigma(s_1 \mid \omega_1) = \sigma(s_2 \mid \omega_2) \equiv \sigma^*. \tag{16}
$$

Note that when $\sigma^* = \frac{1}{2}$, communication is uninformative, since the implied posteriors equal the prior: $\mu_0 = \mu_1 = \mu_2$. The main result is summarized in Proposition 1, with a full derivation provided in the Appendix.

**Proposition 1.** *Under Assumptions 1–4, the central bank's optimal information design distinguishes two cases:*

1. *If* $2\omega \geq \gamma\nu$*, the central bank communicates uninformatively:* $\sigma^* = \frac{1}{2}$.
2. *If* $2\omega < \gamma\nu$*, the central bank communicates informatively:* $\sigma^* \in \left(\frac{1}{2}, 1\right)$*, where* $\sigma^*$ *solves:*

$$1 + \left.\frac{\mathrm{d}\ln(1 - \delta c(\sigma))}{\mathrm{d}\sigma}\right|_{\sigma=\sigma^*} \left(\sigma^* - \frac{1}{2}\right) = 0. \tag{17}$$

Proposition 1 provides significant insight into central bank communication. First, the extensive margin—whether the central bank chooses to communicate informatively or not—depends on whether there is scope for communication to stabilize the economy by guiding individual inflation expectations. Recall that $\gamma$ denotes the sensitivity of the unemployment gap to inflation surprises, while monetary flexibility $\nu$ determines how effectively the central bank can influence inflation expectations by shaping individual beliefs, as shown in Equation (15). The effectiveness of central bank communication is therefore governed by the joint effect of $\gamma$ and $\nu$, relative to the magnitude of the fundamental employment shock $\omega$.

When the shock is overwhelming—such that its magnitude exceeds what monetary policy can feasibly offset, i.e., $2\omega \geq \gamma\nu$—the central bank opts for uninformative communication in order to fully exploit inflation surprises to counteract the shock. In contrast, when there is room for communication-based intervention, i.e., $2\omega < \gamma\nu$, the central bank communicates informatively to complement monetary policy by avoiding excessive inflation surprises.

Second, when informative communication is preferred, the intensive margin—that is, the precision of information—is constrained by the cost of processing information faced by individuals. Recall that $1 - \delta c(\sigma)$ denotes the share of inattentive individuals who pay attention to central bank communication for a given information structure $\sigma$. The derivative $\frac{\mathrm{d}\ln(1-\delta c(\sigma))}{\mathrm{d}\sigma} \leq 0$ captures the rate at which inattentive individuals disengage from communication as informational complexity increases. This rate depends on the share of inattentive individuals: when $\delta$ is larger, disengagement is more sensitive to increases in complexity, causing the rate to decline more steeply.

To satisfy the condition in Equation (17), the central bank must choose an informative message $\sigma^* > \frac{1}{2}$. However, it faces a fundamental trade-off: while more precise communication improves coordination on inflation expectations, it also imposes higher cognitive costs on inattentive individuals, thereby reducing overall reach. The optimal degree of informativeness thus balances informational precision with popularity.

**Discussion** Our benchmark results highlight the critical role of central bank communication in macroeconomic stabilization. During moderate periods, the central bank can strategically deploy informative communication to attract attention and guide inflation expectations, thereby stabilizing the economy without frequently adjusting the underlying monetary policy rule.

By contrast, in times of severe distress—when the economy is hit by an unprecedented adverse shock (i.e., $\omega \gg 0$)—it may be optimal for the central bank to remain deliberately vague. In such cases, uninformative communication can amplify inflation surprises, helping to absorb the shock and mitigate unemployment fluctuations.

Moreover, when the Phillips curve flattens (i.e., $\gamma$ is small), the effectiveness of inflation surprises declines. In response, the central bank may need to resort to unconventional monetary tools to enhance its flexibility (i.e., increase $\nu$), thereby improving its ability to influence expectations through communication.

# 4 Communication under Belief Heterogeneity

We explore the role of belief heterogeneity in central bank communication by relaxing Assumption 1, which imposes homogeneous and neutral prior beliefs for both individuals and the central bank. To this end, we numerically solve for the optimal communication strategy defined in Equation (14), allowing for heterogeneous priors. Section 4.1 introduces the benchmark parameterization. In Section 4.2, we conduct comparative statics to analyze how differences in beliefs affect the central bank's optimal communication policy and its macroeconomic implications.

## 4.1 Benchmark Parameterization

The share of inattentive individuals is set to $\delta = 1$. The shock magnitudes are symmetric, with $\omega_1 = 1$ and $\omega_2 = -1$, representing deviations of one percentage point in the unemployment rate from its natural level. Both individuals and the central bank are assumed to hold homogeneous and neutral prior beliefs, with $\mu_0 = \mu_0^c = 1/2$. The inflation sensitivity parameter is set to $\gamma = 2.94$, calibrated to match the estimate of $\psi = 0.34$ in Hazell, Herreño, Nakamura, and Steinsson

(2022).[6] The central bank's inflation target is fixed at $\pi^T = 2$, reflecting the standard 2% target used in practice. The monetary policy parameter is set to $\nu = 1$, indicating that the central bank tolerates a 1 percentage point deviation from the target to stabilize the economy. Table 1 summarizes the benchmark parameter values used in the analysis.

| $\delta$ | $\omega_1$ | $\omega_2$ | $\mu_0$ | $\mu_0^c$ | $\gamma$ | $\pi^T$ | $\nu$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | -1 | 1/2 | 1/2 | 2.94 | 2 | 1 |

Table 1: Benchmark Parameterization

Note that our benchmark parameterization falls into the second case of Proposition 1, in which the central bank communicates informatively while trading off message precision for popularity. In the subsequent analysis, we also consider a counterfactual scenario with a flattened Phillips curve, setting $\gamma = 1$. This case corresponds to the first regime in Proposition 1, where the central bank optimally remains deliberately vague. By comparing these two regimes, we are able to examine how the optimal communication strategy varies along both the intensive and extensive margins.

## 4.2 Heterogeneous Beliefs

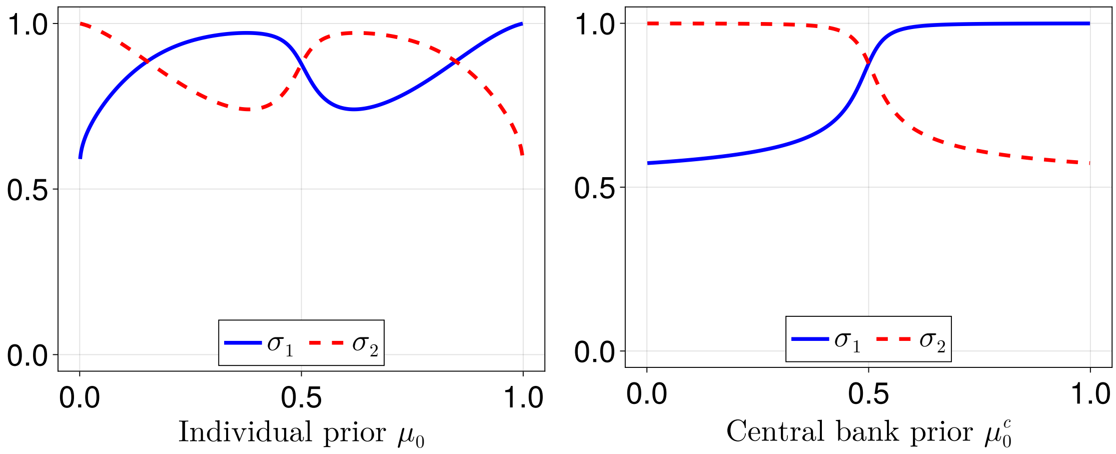
Prior beliefs play a crucial role in shaping the central bank's communication strategy. In our benchmark case, individuals and the central bank share identical priors, each assigning equal probabilities to the two states $\omega_1$ and $\omega_2$, i.e., $\mu_0 = \mu_0^c = 1/2$. Recall that $\mu_0$ and $\mu_0^c$ represent the beliefs of individuals and the central bank, respectively, about the economy being in state $\omega_1$, which corresponds to a high-unemployment scenario. Thus, $\mu_0 \to 0$ or $\mu_0^c \to 0$ reflects increasing optimism (a stronger belief in the strong state $\omega_2$), while $\mu_0 \to 1$ or $\mu_0^c \to 1$ indicates rising pessimism (a stronger belief in the weak state $\omega_1$).

To examine how belief heterogeneity affects the optimal communication strategy, we consider two scenarios: one in which individuals' priors vary from 0 to 1 while holding the central bank's prior fixed, and another in which the central bank's prior varies while keeping the individuals' prior fixed. The resulting optimal message precision, $(\sigma_1, \sigma_2)$, is shown in Figure 1, where $\sigma_1 = \sigma(s_1 \mid \omega_1)$ and

[6] In our framework, $\gamma$ corresponds to the reciprocal of the Phillips curve slope parameter $\psi$ in Hazell et al. (2022), i.e., $\gamma = 1/\psi$.

Figure 1: Optimal Message Precision Across Heterogeneous Prior Beliefs

(a) Individuals (b) Central Bank




$\sigma_2 = \sigma(s_2 \mid \omega_2)$. Panel (a) presents the case with heterogeneous individual priors, and Panel (b) displays the case with heterogeneous central bank priors.

As Figure 1a shows, both individual optimism and pessimism lead to deviations from the benchmark information design. Under the benchmark parameterization, inflation surprises are excessive, prompting the central bank to prefer informative communication as a tool for stabilization. However, due to the cognitive cost of information processing faced by inattentive individuals, the central bank opts for a symmetric yet moderately informative message structure—choosing precisions that are close to but below one. This reflects a deliberate trade-off between precision and popularity.

When individuals are mildly pessimistic (e.g., $\mu_0 = 0.6$), the central bank chooses $\sigma_2 \approx 1$ and $\sigma_1 < 1$. This is because pessimistic beliefs imply that individuals perceive the weak state to be more likely ex-ante. Consequently, their inflation expectations are more closely aligned with actual inflation in the weak state, resulting in smaller inflation surprises. In contrast, the strong state becomes relatively unexpected, amplifying inflation surprises. To manage these larger surprises in the strong state while preserving the popularity of its communication—given the cost of information processing—the central bank optimally trades off precision in the weak state to increase the likelihood of message $s_2$, which allows to mitigate excessive inflation surprises and improve coordination under the strong state.

However, when individuals perceive the weak state to occur almost surely (e.g., $\mu_0 \approx 1$), the central bank chooses $\sigma_1 \approx 1$ and $\sigma_2 < 1$. Recall from Equation (9) that the cost of processing information depends on the distance between priors and posteriors. With growing pessimism, the cost of guiding individuals toward posterior beliefs favoring the strong state increases sharply, whereas the cost of confirming beliefs about the weak state declines toward zero. This informational cost channel dominates the earlier strategic trade-off concerning inflation surprises under the strong state. As a result, the central bank finds it more effective to allocate its communication precision to confirming the weak state rather than correcting expectations toward the less likely strong state.

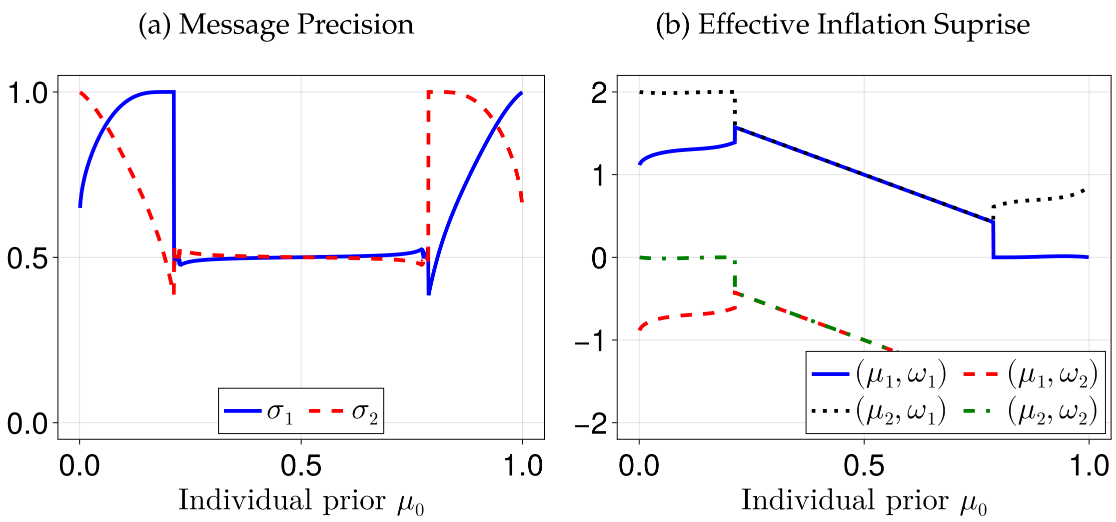
The same logic applies in the case of optimism, leading to an opposite pattern as individual beliefs become more optimistic in Figure 1a. Moreover, the central bank's intervention is rarely fully informative for most individual prior configurations due to the presence of inattentive individuals. A decrease in the share of inattentive individuals (i.e., $\delta = 0.5$) predictably increases the overall precision of the central bank's messages. This effect is asymmetric when individual beliefs deviate from neutrality. For illustration, see Figure 7 in the Appendix B.

Figure 1b illustrates the effect of central bank priors on its communication strategy. Recall that in Equation (14), the central bank minimizes expected losses across states using its own prior beliefs as weights. In other words, the central bank's prior determines the relative importance it places on stabilizing each state. This prior can be interpreted as reflecting the central bank's superior knowledge or informed judgment about the likelihood of different economic conditions. Based on this belief, the central bank decides whether and how to construct informative messages to guide individuals—who may hold different priors—in order to stabilize the economy.

When the central bank becomes more pessimistic ($\mu_0^c \to 1$), it increases the precision of messages related to the more plausible weak state while reducing precision for the less likely strong state. The opposite holds when the central bank is more optimistic ($\mu_0^c \to 0$). Essentially, the central bank strategically sacrifices message precision for the less likely state in order to enhance precision for the more probable one. This trade-off reflects the central bank's effort to minimize expected losses in the state it perceives as more likely, while still preserving communication popularity among inattentive individuals.

Nevertheless, due to the presence of inattentive individuals, the central bank cannot fully eliminate inflation surprises. As before, this constraint becomes less binding when the share of inattentive individuals decreases (i.e., $\delta = 0.5$). For

Figure 2: Optimal Communication by Individual Priors with a Flat Phillips Curve




further illustration, refer to Figure 7 in the Appendix B.

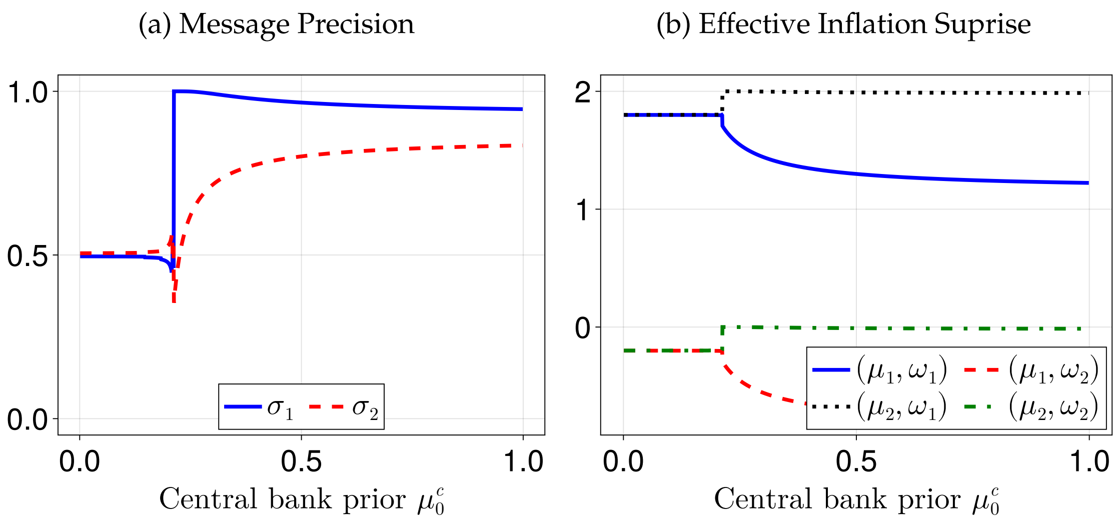
**Flatter Phillips Curve** It has been shown that the Phillips curve has flattened over time, meaning that unemployment has become less responsive to inflation surprises. In the context of our model, this corresponds to a decline in the sensitivity parameter $\gamma$ in Equation (1). This structural shift raises concerns for policymakers regarding the diminishing effectiveness of monetary policy, particularly in its role of shaping market participants' inflation expectations.

Under the benchmark parameterization, setting $\gamma = 1$ satisfies the inequality $2\omega \geq \gamma\nu$ in Proposition 1, implying that the magnitude of the employment shock exceeds the counteracting effect of inflation surprises. Consequently, in the symmetric case, the central bank finds it optimal to provide uninformative messages. This strategy allows the central bank to fully exploit inflation surprises to mitigate unemployment fluctuations, as the power of expectation management weakens in a flatter Phillips curve environment.

A natural question then arises: how does this result extend when individuals and the central bank hold heterogeneous beliefs? To address this, we revisit the optimal central bank communication under belief heterogeneity when $\gamma = 1$.

In Figure 2, we present the optimal communication strategy under a flattened Phillips curve of $\gamma = 1$. Panel (a) shows the precision of the central bank's messages, $(\sigma_1, \sigma_2)$, as individual priors $\mu_0$ vary from 0 to 1. Panel (b) displays the corresponding effective inflation surprises, measured as $\gamma\left(\pi(\omega) - \pi^e(\mu_j)\right)$ for

Figure 3: Optimal Communication with Optimistic Individuals by Central Bank Priors with a Flat Phillips Curve




each posterior belief $\mu_j$ and state $\omega$.

Different from the benchmark, the central bank provides information if individuals are either too optimistic or too pessimistic. In particular, in our benchmark where $\mu_0 = \mu_0^c = \frac{1}{2}$, the inflation surprise exactly compensates for the shocks, thus implying a zero unemployment gap. This is the best scenario for the central bank, and there is no reason to intervene. Consider, for instance, the possibility that individuals are instead too pessimistic (that is, $\mu_0 \to 1$). There exists a threshold value of $\mu_0$ such that above it, the central bank finds it optimal to send informative messages. In particular, above such threshold, $\sigma_2 = 1$, implying that the central bank commits to reveal to individuals whenever the negative shock (i.e., $\omega_1$) realizes. Why? Note that, as $\mu_0 \to 1$, the inflation surprise becomes excessive in the case of a positive shock (i.e., $\omega_2$), whereas it is insufficient in the event of a negative shock. When this imbalance grows excessively, the central bank corrects it using the message $s_2$, which brings the inflation surprise closer to the optimal level. This comes at the cost of increasing the loss associated with $s_1$. Nevertheless, this is optimal for the central bank because $s_2$ occurs more often than $s_1$. Generalizing, the central bank responds to excessive optimism or pessimism by individuals inducing the optimal inflation surprise when the shock that individuals consider the least plausible realizes.

Instead, superior knowledge by the central bank has no impact on information design. See Figure 8 in Appendix B. As mentioned before, the reason is that inflation surprise is already at its optimal level given $\mu_0 = \frac{1}{2}$. Thus, the central bank finds it optimal to provide no further information. However, the central bank's private information plays a role when individuals are optimistic or pessimistic. See Figure 3, where we assume that $\mu_0 = 0.1$, meaning that individuals are optimistic. The central bank intervenes when it does not share such optimism: when $\mu_0^c$ is above a threshold, the central bank sends informative messages. The central bank uses the message $s_1$ to return inflation surprise to its optimal value. This manipulation requires using message $s_2$ to reveal when the economy is strong (i.e., $\omega_2$), which, however, comes with little surprise given individuals' prior beliefs and makes this information design optimal. Therefore, our analysis shows that the central bank provides information when beliefs are sufficiently heterogeneous.

Remarkably, inattention is again a constraint for the central bank when it decides to communicate. As before, decreasing the share of inattentive individuals (i.e., $\delta = 0.5$) increases the precision of the central bank's messages. See Figures 9 and 10 in Appendix B.

# 5 Central Bank Voice under Turbulence

We examine the role of asymmetric employment shocks in central bank communication by relaxing Assumption 2. Proposition 1 shows that, in a setting with homogeneous prior beliefs and symmetric shocks, there exists a threshold value in the magnitude of the shock such that the central bank provides information if $2\omega < \gamma\nu$, whereas it does not otherwise. Introducing asymmetric shocks alone does not change the prediction: the central bank communicates if the aggregate magnitude of the shocks (i.e., $|\omega_1| + |\omega_2|$) is relatively small.

Then, we study how asymmetric shocks come into play with belief heterogeneity, particularly the roles of shock magnitude and asymmetry. To this end, in contrast to the benchmark, where we consider $(\omega_1, \omega_2) = (1, -1)$, we further examine the effect of belief heterogeneity in two extra cases: $(\omega_1, \omega_2) = (2, -2)$ and $(\omega_1, \omega_2) = (2, -1)$. The first case aims to study the role of shock magnitude, while the second checks the role of shock asymmetry. We redo the previous exercises with these different shock combinations and follow the same convention for reporting results. The results are that neither shocks magnitude nor their asymmetry have an effect on information design. See Figures 11-14 in Appendix B. The intuition is that, even with belief heterogeneity, since the aggregate magnitude of the shocks is relatively small, the central bank communicates and thus inattention is the main constraint, as already shown in Section 4.2.

**Flatter Phillips Curve** In this scenario, the magnitude and asymmetry of unemployment shocks affect the central bank's information design because they impact the optimality of inflation surprise in the benchmark. Let us begin with $(\omega_1, \omega_2) = (2, -2)$. In this case, when $\mu_0 = \frac{1}{2}$, the inflation surprise is insufficient, independently of the shock. Varying $\mu_0$ brings inflation surprise closer to the optimal level for one shock, but meanwhile away from the optimal level for the other shock. Information design cannot help the central bank in this scenario. See Figures 15-16 in Appendix B. Instead, when shocks are asymmetric i.e., $(\omega_1, \omega_2) = (2, -1)$, there are two changes. First, increasing the magnitude of the negative shock $\omega_1$ makes the inflation surprise optimal even for extremely optimistic individuals' beliefs. Second, even if the magnitude of the positive shock is the same, individuals must be more pessimistic than before for the central bank to find it optimal to communicate. See Figure 17 in Appendix B.

# 6 Talking with Skewed, Behavioral Receivers

We study the role of adaptive expectation and the distribution of attention budget in central bank communication by relaxing Assumptions 3 and 4.

## 6.1 Skewed individuals

In this section, we relax Assumption 4. In particular, we consider an alternative distribution for the attention budget such that many individuals have a low attention budget, i.e., a right-skewed distribution of attention budget, and we analyze the impact of lower attention compared to the case where attention is uniformly distributed. In particular, we use the Kumaraswamy distribution, whose probability density function takes the following form:

$$f(x; a, b) = a \cdot b \cdot x^{a-1} \cdot (1 - x^a)^{b-1} \tag{18}$$

$$F(x; a, b) = 1 - (1 - x^a)^b \tag{19}$$

where $a$ and $b$ are non-negative shape parameters. This distribution collapses to a uniform distribution when $a = b = 1$ and becomes right-skewed with $a = 2$ and $b = 5$. When considering the symmetric benchmark (Section 3), the results are qualitatively similar. The only difference concerns the precision of information when shocks are weak, as the next proposition shows.

**Proposition 2.** *Under Assumptions 1–3, the central bank's optimal information design distinguishes two cases:*

1. *If $2\omega \geq \gamma\nu$, the central bank communicates uninformatively: $\sigma^* = \frac{1}{2}$.*
2. *If $2\omega < \gamma\nu$, the central bank communicates informatively: $\sigma^* \in \left(\frac{1}{2}, 1\right)$, where $\sigma^*$ solves:*

$$1 + \left( \frac{\boldsymbol{d} \ln \left(1 - \delta \left(1 - [c(\sigma)]^a\right)^b\right)}{\boldsymbol{d}\sigma} \Bigg|_{\sigma=\sigma^*} \right) \left(\sigma^* - \frac{1}{2}\right) = 0 \tag{20}$$

However, changing the distribution of attention has a more significant effect when we consider asymmetric scenarios (Section 4). In particular, we investigate how the shape parameters affect the effectiveness of central bank communication, e.g., $\partial\sigma/\partial a$. Our results show that the central bank's optimal communication policy crucially depends on the share of inattentive individuals.

We consider the case where $\gamma = 2.94$. When $\delta = 1$ (i.e., all individuals are inattentive), intuitively, the lower the attention budget across individuals, the less precise the information provided by the central bank. In particular, compared to the uniform benchmark $(a, b) = (1, 1)$, when $(a, b) = (2, 5)$—more individuals have low attention budgets—the central bank sends less informative messages, whereas when $(a, b) = (5, 2)$—more individuals have high attention budgets—the central bank sends more informative messages. See Figures 1a and 4.

The results when $\delta = 0.5$ (i.e., half of the individuals are inattentive) are visualized in Figure 5. our findings are instead counterintuitive. Compared to the uniform benchmark in Figure 7 in Appendix B, when $(a, b) = (2, 5)$, the central bank simply gives up talking with inattentive individuals but sends the most informative messages to attentive individuals (those without information processing cost). In this sense, the central bank gets the attentive half perfectly informed. By contrast, when $(a, b) = (5, 2)$, the central bank sends less informative messages to get the attention of almost all individuals.

Figure 4: Optimal Message Precision across Individual Priors with Skewed Attention Budgets

(a) Right-Skewed $(a, b) = (2, 5)$ (b) Left-Skewed$(a, b) = (5, 2)$

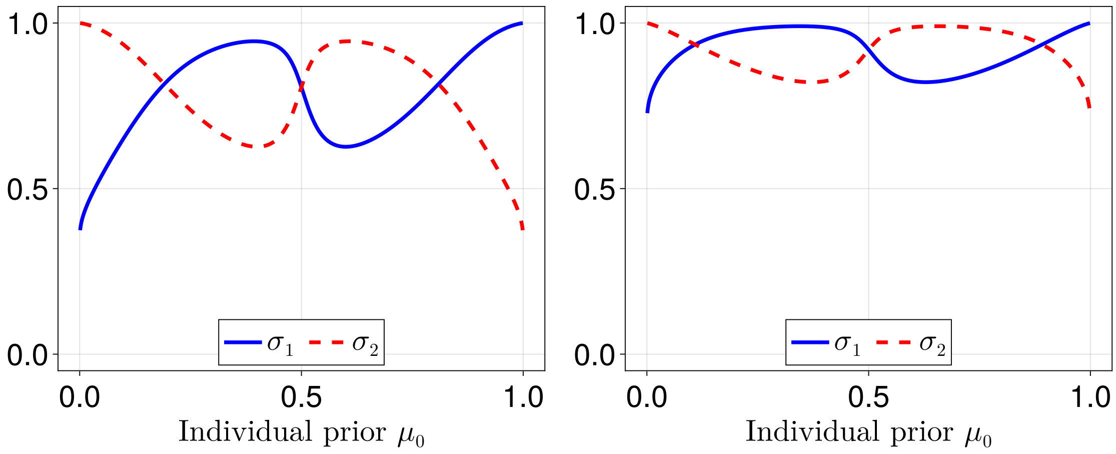
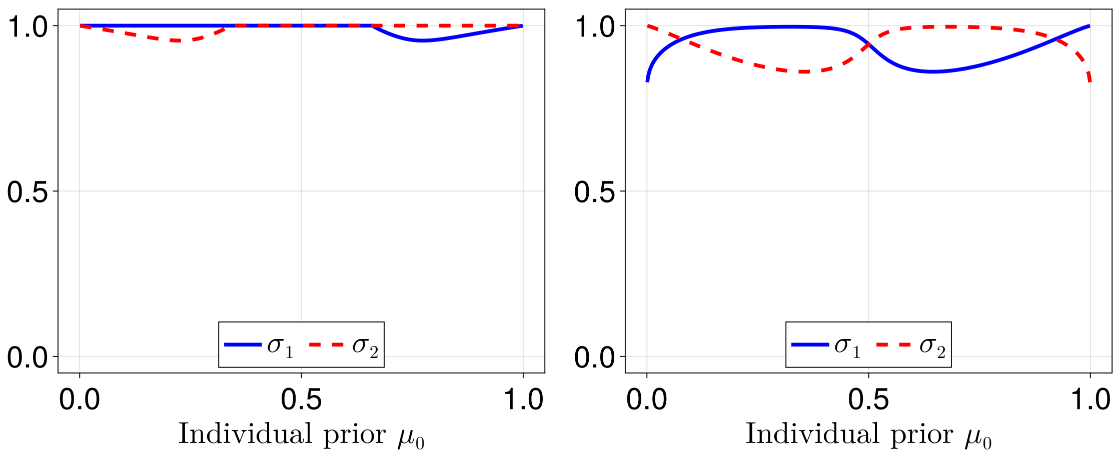



Figure 5: Optimal Message Precision across Individual Priors with Half Inattentive Individuals of Skewed Attention Budgets

(a) Right-Skewed $(a, b) = (2, 5)$ (b) Left-Skewed$(a, b) = (5, 2)$




We find qualitatively similar results when $\gamma = 1$, although only when the difference in prior beliefs between the central bank and individuals justifies the central bank's information provision. See Figures 18 and 19. Overall, we find that the central bank takes into account the share of inattentive individuals and the distribution of attention among inattentive individuals when strategically formulating its communication.

## 6.2 Irrational Inflation Expectations

In this section, we relax Assumption 3. In particular, we assume that beliefs are adaptive:

$$\hat{\mu} = \mu_0 + \theta(\mu - \mu_0) = (1-\theta)\mu_0 + \theta\mu,$$

where $\theta \in [0,1]$.

**Lemma 1.** *Adaptive belief and adaptive expectation are equivalent given the linear inflation expectation function $\pi^e$, where $\theta$ governs the extent to which information is used.*

Under the symmetric benchmark—i.e., Assumptions 1 and 2 hold—the results do not differ qualitatively, as the next proposition shows.

**Proposition 3.** *Under Assumptions 1–2, the central bank's optimal information design distinguishes two cases:*

1. *If $2\omega \geq \gamma\nu(2-\theta)$, the central bank communicates uninformatively: $\sigma^* = \frac{1}{2}$.*
2. *If $2\omega < \gamma\nu(2-\theta)$, the central bank communicates informatively: $\sigma^* \in \left(\frac{1}{2}, 1\right)$, where $\sigma^*$ solves:*

$$1 + \left(\frac{\boldsymbol{d} \ln(1-\delta F(c(\sigma)))}{\boldsymbol{d}\sigma}\right)\left(\sigma^* - \frac{1}{2}\right) = 0. \tag{21}$$

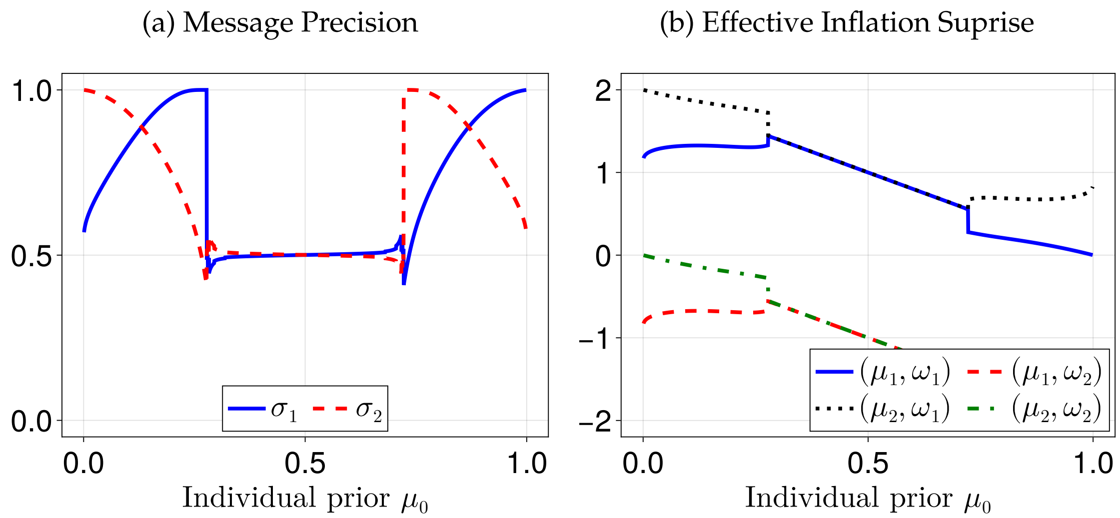
When shocks are strong (i.e., $2\omega \geq \gamma\nu(2-\theta)$), the central bank designs uninformative messages. Otherwise, the central bank designs partially or fully informative messages—the optimal precision of information solves equation (21) for a generic distribution of attention $F(\cdot)$. Proposition 3 shows that the central bank is more likely to talk informatively with adaptive individuals (i.e., when $\theta < 1$). The central bank provides information more frequently to overcome the updating loss in the adaptive expectation process, narrowing the range of uninformative communication. In particular, the comparison between Proposition 1 and Proposition 3 shows that in the presence of irrational inflation expectations, the central bank communicates even in the event of shocks of intermediate magnitude. Table 2 summarizes the comparison.

When considering asymmetric scenarios, we additionally find that introducing irrational inflation expectations makes the central bank prone to communicate when individuals are moderately too pessimistic or optimistic. Indeed, the threshold values of $\mu_0$, which make it optimal for the central bank to communi-

|  | Inflation expectations |  |
|---|---|---|
| Shock magnitude: $\omega$ | Rational: $\theta = 1$ | Irrational: $\theta \in (0,1)$ |
| $\omega \in [0, \frac{\gamma\nu}{2}]$ | $\sigma^* \in \left(\frac{1}{2}, 1\right)$ | $\sigma^* \in \left(\frac{1}{2}, 1\right)$ |
| $\omega \in [\frac{\gamma\nu}{2}, \frac{\gamma\nu(2-\theta)}{2}]$ | $\sigma^* = \frac{1}{2}$ | $\sigma^* \in \left(\frac{1}{2}, 1\right)$ |
| $\omega \in [\frac{\gamma\nu(2-\theta)}{2}, \infty)$ | $\sigma^* = \frac{1}{2}$ | $\sigma^* = \frac{1}{2}$ |

Table 2: Optimal information design by the central bank in symmetric scenarios as a function of shock magnitude and inflation expectations.

Figure 6: Optimal Communication by Individual Priors with a Flat Phillips Curve and Adaptive Individuals




cate even if $\gamma = 1$ (i.e., shocks are relatively strong), are closer to $\frac{1}{2}$, as can be seen by comparing Figure 6 with $\theta = 0.5$ with Figure 2 with $\theta = 1.0$.

# 7 Conclusion

This paper studies the optimal information design of a central bank with commitment. We show that strategically crafted communication can serve as a powerful policy instrument for macroeconomic stabilization. Specifically, the central bank adjusts its information provision depending on whether the inflation surprise is excessive relative to the underlying unemployment shock. When inflation surprises are needed to counteract large shocks, the central bank optimally remains deliberately vague. Otherwise, it communicates informatively, balancing message precision against popularity to manage individual inflation expectations more effectively.

When individuals are either overly optimistic or pessimistic, inflation surprises tend to intensify in the state they consider less likely. In response, the central bank adjusts its communication strategy by providing a more informative message about the less plausible state to better align expectations with actual inflation and mitigate excessive surprises. In contrast, when the central bank holds asymmetric beliefs about the states, it strategically targets precision in the state it deems more probable to minimize expected losses.

We also demonstrate how shock asymmetry, the distribution of attention budgets, and adaptive expectation formation shape the optimal communication strategy. In essence, the optimal policy hinges on a trade-off between message precision and popularity, as well as macroeconomic fundamentals and individual characteristics.

Our results offer a novel interpretation of deliberately vague communication, such as Mario Draghi's "whatever it takes" statement. During a period of severe economic distress, the ECB faced a substantial unemployment shock. In such a context, preserving inflation surprises through vagueness allowed the central bank to stabilize the economy more effectively. In contrast, fully revealing the state of the economy would have neutralized the inflation surprise, reducing its capacity to counteract the shock.

Finally, we leave the study of coordinated design between monetary and communication policies for future research. As Proposition 1 illustrates, a central bank could in principle optimize both simultaneously to enhance stabilization in the face of economic fluctuations.

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# A Mathematical Details

We denote with $\sigma_1 = \sigma(s_1|\omega_1)$ and $\sigma_2 = \sigma(s_2|\omega_2)$. It follows that:

$$\mu_1 = \frac{\sigma_1\mu_0}{\sigma_1\mu_0 + (1-\sigma_2)(1-\mu_0)} \tag{22}$$

$$\mu_2 = \frac{(1-\sigma_1)\mu_0}{(1-\sigma_1)\mu_0 + \sigma_2(1-\mu_0)} \tag{23}$$

$$\frac{\partial\mu_1}{\partial\sigma_1} = \frac{\mu_0(1-\mu_0)(1-\sigma_2)}{[\sigma_1\mu_0 + (1-\sigma_2)(1-\mu_0)]^2} \tag{24}$$

$$\frac{\partial\mu_2}{\partial\sigma_1} = \frac{-\mu_0(1-\mu_0)\sigma_2}{[(1-\sigma_1)\mu_0 + \sigma_2(1-\mu_0)]^2} \tag{25}$$

$$\frac{\partial\mu_1}{\partial\sigma_2} = \frac{\mu_0(1-\mu_0)\sigma_1}{[\sigma_1\mu_0 + (1-\sigma_2)(1-\mu_0)]^2} \tag{26}$$

$$\frac{\partial\mu_2}{\partial\sigma_2} = \frac{-\mu_0(1-\mu_0)(1-\sigma_1)}{[(1-\sigma_1)\mu_0 + \sigma_2(1-\mu_0)]^2} \tag{27}$$

$$\tau_1 = \sigma_1\mu_0 + (1-\sigma_2)(1-\mu_0) \tag{28}$$

$$\tau_2 = (1-\sigma_1)\mu_0 + \sigma_2(1-\mu_0) \tag{29}$$

$$\frac{\partial\tau_j}{\partial\sigma_1} = \begin{cases} \mu_0 & \text{if } j = 1 \\ -\mu_0 & \text{otherwise} \end{cases} \tag{30}$$

$$\frac{\partial\tau_j}{\partial\sigma_2} = \begin{cases} 1-\mu_0 & \text{if } j = 2 \\ -(1-\mu_0) & \text{otherwise} \end{cases} \tag{31}$$

$$\frac{\partial H(\mu_j)}{\partial\sigma_k} = -\frac{\partial\mu_j}{\partial\sigma_k}\ln\left(\frac{\mu_j}{1-\mu_j}\right) \tag{32}$$

$$c_k'(\sigma) = -\chi\sum_{j=1,2}\left[\frac{\partial\tau_j}{\partial\sigma_k}H(\mu_j) + \tau_j\frac{\partial H(\mu_j)}{\partial\sigma_k}\right] \tag{33}$$

**Proof of Proposition 1** Under Assumption 3, the problem in (14) becomes:

$$\min_{\sigma}\ \delta F(c(\sigma))\left\{\mu_0^c\left[\omega_1 - 2\gamma\nu(1-\mu_0)\right]^2 + (1-\mu_0^c)\left[\omega_2 + 2\gamma\nu\mu_0\right]^2\right\} +$$
$$+ [1-\delta F(c(\sigma))]\left\{\mu_0^c\left[\sigma_1(\omega_1 - 2\gamma\nu(1-\mu_1))^2 + (1-\sigma_1)(\omega_1 - 2\gamma\nu(1-\mu_2))^2\right] +\right.$$
$$\left. + (1-\mu_0^c)\left[\sigma_2(\omega_2 + 2\gamma\nu\mu_2)^2 + (1-\sigma_2)(\omega_2 + 2\gamma\nu\mu_1)^2\right]\right\}$$

The F.O.C. are:

$$\delta f(c(\sigma))c_1'(\sigma)\left\{\mu_0^c\left[\omega_1-2\gamma\nu(1-\mu_0)\right]^2+(1-\mu_0^c)\left[\omega_2+2\gamma\nu\mu_0\right]^2\right\}+$$
$$-\delta f(c(\sigma))c_1'(\sigma)\left\{\mu_0^c\left[\sigma_1(\omega_1-2\gamma\nu(1-\mu_1))^2+(1-\sigma_1)(\omega_1-2\gamma\nu(1-\mu_2))^2\right]+\right.$$
$$\left.+(1-\mu_0^c)\left[\sigma_2(\omega_2+2\gamma\nu\mu_2)^2+(1-\sigma_2)(\omega_2+2\gamma\nu\mu_1)^2\right]\right\}+$$
$$+[1-\delta F(c(\sigma))]\left\{\mu_0^c\left[(\omega_1-2\gamma\nu(1-\mu_1))^2+4\sigma_1(\omega_1-2\gamma\nu(1-\mu_1))\gamma\nu\frac{\partial\mu_1}{\partial\sigma_1}+\right.\right.$$
$$\left.-(\omega_1-2\gamma\nu(1-\mu_2))^2+4(1-\sigma_1)(\omega_1-2\gamma\nu(1-\mu_2))\gamma\nu\frac{\partial\mu_2}{\partial\sigma_1}\right]+$$
$$\left.+4\gamma\nu(1-\mu_0^c)\left[\sigma_2(\omega_2+2\gamma\nu\mu_2)\frac{\partial\mu_2}{\partial\sigma_1}+(1-\sigma_2)(\omega_2+2\gamma\nu\mu_1)\frac{\partial\mu_1}{\partial\sigma_1}\right]\right\}=0$$

$$\delta f(c(\sigma))c_2'(\sigma)\left\{\mu_0^c\left[\omega_1-2\gamma\nu(1-\mu_0)\right]^2+(1-\mu_0^c)\left[\omega_2+2\gamma\nu\mu_0\right]^2\right\}+$$
$$-\delta f(c(\sigma))c_2'(\sigma)\left\{\mu_0^c\left[\sigma_1(\omega_1-2\gamma\nu(1-\mu_1))^2+(1-\sigma_1)(\omega_1-2\gamma\nu(1-\mu_2))^2\right]+\right.$$
$$\left.+(1-\mu_0^c)\left[\sigma_2(\omega_2+2\gamma\nu\mu_2)^2+(1-\sigma_2)(\omega_2+2\gamma\nu\mu_1)^2\right]\right\}+$$
$$+[1-\delta F(c(\sigma))]\left\{(1-\mu_0^c)\left[(\omega_2+2\gamma\nu\mu_2)^2+4\sigma_2(\omega_2+2\gamma\nu\mu_2)\gamma\nu\frac{\partial\mu_2}{\partial\sigma_2}+\right.\right.$$
$$\left.-(\omega_2+2\gamma\nu\mu_1)^2+4(1-\sigma_2)(\omega_2+2\gamma\nu\mu_1)\gamma\nu\frac{\partial\mu_1}{\partial\sigma_2}\right]+$$
$$\left.+4\gamma\nu\mu_0^c\left[\sigma_1(\omega_1-2\gamma\nu(1-\mu_1))\frac{\partial\mu_1}{\partial\sigma_2}+(1-\sigma_1)(\omega_1-2\gamma\nu(1-\mu_2))\frac{\partial\mu_2}{\partial\sigma_2}\right]\right\}=0$$

Under Assumptions 1-2, the F.O.C.s becomes symmetric. Thus, we consider symmetric solutions where $\sigma_1=\sigma_2=\sigma^*$ and $\sigma^*\in\left[\frac{1}{2},1\right]$ is the precision of the central bank's information. It follows that the solution to the central bank's information design problem solves the following condition:

$$2\gamma\nu(4\omega-2\gamma\nu)\left\{[1-\delta F(c(\sigma))](2\sigma^*-1)+2\delta f(c(\sigma))c'(\sigma)\left[\sigma^*(1-\sigma^*)-\frac{1}{4}\right]\right\}=0$$

Removing irrelevant positive terms yields:

$$(2\omega - \gamma\nu)\left\{[1-\delta F(c(\sigma))]\left(\sigma^* - \frac{1}{2}\right) + \delta f(c(\sigma))c'(\sigma)\left[\sigma^*(1-\sigma^*) - \frac{1}{4}\right]\right\} = 0$$

Observe that:

$$\sigma^*(1-\sigma^*) - \frac{1}{4} = -\left(\sigma^* - \frac{1}{2}\right)^2$$

and that $1 - \delta F(c(\sigma)) > 0$ for any $\sigma$ and $\delta \in [0,1]$. Substituting the quadratic expression and dividing by $1 - F(c(\sigma))$ yields:

$$(2\omega - \gamma\nu)\left\{\left(\sigma^* - \frac{1}{2}\right) - \frac{\delta f(c(\sigma))}{1-\delta F(c(\sigma))}c'(\sigma)\left(\sigma^* - \frac{1}{2}\right)^2\right\} = 0$$

Observe that:

$$\frac{\boldsymbol{d} \ln(1-\delta F(c(\sigma)))}{\boldsymbol{d}\sigma} = -\frac{\delta f(c(\sigma))}{1-\delta F(c(\sigma))}c'(\sigma) \leq 0$$

This implies that:

$$(2\omega - \gamma\nu)\left(\sigma^* - \frac{1}{2}\right)\left[1 + \left(\frac{\boldsymbol{d} \ln(1-\delta F(c(\sigma)))}{\boldsymbol{d}\sigma}\right)\left(\sigma^* - \frac{1}{2}\right)\right] = 0$$

Under Assumption 4, $F$ denotes the CDF for a uniform distribution on $[0,1]$:

$$F(c(\sigma)) = c(\sigma)$$

This implies:

$$\ln(1-\delta F(c(\sigma))) = \ln\left(1 - \delta c(\sigma)\right)$$

Accordingly, two possible candidates satisfy the necessary condition: $\underline{\sigma} = \frac{1}{2}$ and $\overline{\sigma} \in (\frac{1}{2}, 1)$ such that:

$$1 + \left(\left.\frac{\boldsymbol{d} \ln(1-\delta c(\sigma))}{\boldsymbol{d}\sigma}\right|_{\sigma=\overline{\sigma}}\right)\left(\overline{\sigma} - \frac{1}{2}\right) = 0$$

Therefore $\sigma^* \in \{\underline{\sigma}, \overline{\sigma}\}$. To show which candidate is sufficient, we compare utilities across them. Central bank's utility with a generic $\widehat{\sigma}$ is:

$$\begin{aligned} u_{\widehat{\sigma}} = &- \delta F(c(\sigma))(\omega - \gamma\nu)^2 \\ &- (1-\delta F(c(\sigma)))\left[\widehat{\sigma}(\omega - 2\gamma\nu(1-\widehat{\sigma}))^2 + (1-\widehat{\sigma})(\omega - 2\gamma\nu\widehat{\sigma})^2\right] \end{aligned}$$

Its utility with uninformative communication $\underline{\sigma}$ is:

$$u_{\underline{\sigma}} = -(\omega - \gamma\nu)^2$$

For $\sigma^* = \underline{\sigma}$ to be optimal, $u_{\underline{\sigma}} - u_{\widehat{\sigma}} \geq 0$. Under which condition does this relationship hold?

In the first step, $u_{\widehat{\sigma}}$ can be rewritten into:

$$\begin{aligned} u_{\widehat{\sigma}} =& -\delta F(c(\sigma))(\omega - \gamma\nu)^2 \\ & -(1 - \delta F(c(\sigma)))(\omega - \gamma\nu)^2 + (1 - \delta F(c(\sigma)))(\omega - \gamma\nu)^2 \\ & -(1 - \delta F(c(\sigma)))\left[\widehat{\sigma}(\omega - 2\gamma\nu(1 - \widehat{\sigma}))^2 + (1 - \widehat{\sigma})(\omega - 2\gamma\nu\widehat{\sigma})^2\right] \end{aligned}$$

It follows that:

$$u_{\widehat{\sigma}} = u_{\underline{\sigma}} - (1 - \delta F(c(\sigma)))\left[\widehat{\sigma}(\omega - 2\gamma\nu(1 - \widehat{\sigma}))^2 + (1 - \widehat{\sigma})(\omega - 2\gamma\nu\widehat{\sigma})^2 - (\omega - \gamma\nu)^2\right]$$

Thus, showing $u_{\underline{\sigma}} - u_{\widehat{\sigma}} \geq 0$ is equivalent to show:

$$\widehat{\sigma}(\omega - 2\gamma\nu(1 - \widehat{\sigma}))^2 + (1 - \widehat{\sigma})(\omega - 2\gamma\nu\widehat{\sigma})^2 - (\omega - \gamma\nu)^2 \geq 0$$

In the second step,

$$\begin{aligned} \widehat{\sigma}(\omega - 2\gamma\nu(1 - \widehat{\sigma}))^2 &= \widehat{\sigma}\omega^2 - 4\omega\gamma\nu(1 - \widehat{\sigma})\widehat{\sigma} + 4\gamma^2\nu^2(1 - \widehat{\sigma})^2\widehat{\sigma} \\ (1 - \widehat{\sigma})(\omega - 2\gamma\nu\widehat{\sigma})^2 &= (1 - \widehat{\sigma})\omega^2 - 4\omega\gamma\nu(1 - \widehat{\sigma})\widehat{\sigma} + 4\gamma^2\nu^2(1 - \widehat{\sigma})\widehat{\sigma}^2 \\ (\omega - \gamma\nu)^2 &= \omega^2 - 2\omega\gamma\nu + \gamma^2\nu^2 \end{aligned}$$

The above inequality can be rearranged as:

$$-8\omega\gamma\nu(1 - \widehat{\sigma})\widehat{\sigma} + 4\gamma^2\nu^2(1 - \widehat{\sigma})\widehat{\sigma} + 2\omega\gamma\nu - \gamma^2\nu^2 \geq 0$$

It can be further expressed nicely as:

$$\begin{aligned} & -4\gamma\nu(1 - \widehat{\sigma})\widehat{\sigma}(2\omega - \gamma\nu) + \gamma\nu(2\omega - \gamma\nu) \geq 0 \\ & \gamma\nu(2\omega - \gamma\nu)\left[1 - 4(1 - \widehat{\sigma})\widehat{\sigma}\right] \geq 0 \end{aligned}$$

Given that $\gamma\nu > 0$ by construction and $1 - 4(1 - \widehat{\sigma})\widehat{\sigma} \geq 0$ for any $\widehat{\sigma} \in [\frac{1}{2}, 1)$, we find that, consistent with the necessary condition, $\sigma^* = \underline{\sigma} = \frac{1}{2}$ if $2\omega - \gamma\nu \geq 0$ while $\sigma^* = \overline{\sigma} \in (\frac{1}{2}, 1)$ if $2\omega - \gamma\nu < 0$.

**Proof of Proposition 2**

*Proof.* The proof is identical to Proposition 1. Indeed, Assumption 4 was not necessary for the above proof. Then, the same argument can be used to prove this proposition. The only difference concerns the condition that identifies $\overline{\sigma}$.

Recall that the CDF for a Kumaraswamy distribution with shape parameters $(a, b)$ is given by:

$$F(c(\sigma); a, b) = 1 - (1 - [c(\sigma)]^a)^b$$

This implies:

$$\ln(1 - \delta F(c(\sigma))) = \ln\left(1 - \delta\,(1 - [c(\sigma)]^a)^b\right)$$

Accordingly, $\overline{\sigma} \in (\frac{1}{2}, 1)$ solves:

$$1 + \left( \left. \frac{\boldsymbol{d} \ln\left(1 - \delta\,(1 - [c(\sigma)]^a)^b\right)}{\boldsymbol{d}\sigma} \right|_{\sigma = \overline{\sigma}} \right) \left( \overline{\sigma} - \frac{1}{2} \right) = 0$$

□

**Proof of Lemma 1**

*Proof.*

$$\begin{aligned}
\pi^e(\hat{\mu}) &= \pi^T - \nu(1 - 2\hat{\mu}) \\
&= \pi^T - \nu\,\{1 - 2[(1-\theta)\mu_0 - \theta\mu]\} \\
&= \pi^T + 2\nu(1-\theta)\mu_0 + 2\nu\theta\mu - \nu \\
&= (1-\theta)\pi^T + \theta\pi^T + 2\nu(1-\theta)\mu_0 + 2\nu\theta\mu - (1-\theta)\nu - \theta\nu \\
&= (1-\theta)\left[\pi^T - \nu(1 - 2\mu_0)\right] + \theta\left[\pi^T - \nu(1 - 2\mu)\right] \\
&= (1-\theta)\pi^e(\mu_0) + \theta\pi^e(\mu) \\
&= \hat{\pi}^e
\end{aligned}$$

□

**Proof of Proposition 3** Under adaptive belief, the problem in (14) becomes:

$$\min_{\sigma}\ \delta F(c(\sigma))\left\{\mu_0^c\left[\omega_1-2\gamma\nu(1-\mu_0)\right]^2+(1-\mu_0^c)\left[\omega_2+2\gamma\nu\mu_0\right]^2\right\}+$$
$$+\left[1-\delta F(c(\sigma))\right]\left\{\mu_0^c\Big[\sigma_1(\omega_1-\gamma\nu\left(1+(1-\theta)(1-2\mu_0)+\theta(1-2\mu_1)\right))^2+\right.$$
$$+(1-\sigma_1)(\omega_1-\gamma\nu\left(1+(1-\theta)(1-2\mu_0)+\theta(1-2\mu_2)\right))^2\Big]+$$
$$+(1-\mu_0^c)\Big[\sigma_2(\omega_2+\gamma\nu\left(1-(1-\theta)(1-2\mu_0)-\theta(1-2\mu_2)\right))^2+$$
$$\left.+(1-\sigma_2)(\omega_2+\gamma\nu\left(1-(1-\theta)(1-2\mu_0)-\theta(1-2\mu_1)\right))^2\Big]\right\}$$

The F.O.C. are:

$$\delta f(c(\sigma))c_1'(\sigma)\left\{\mu_0^c\left[\omega_1-2\gamma\nu(1-\mu_0)\right]^2+(1-\mu_0^c)\left[\omega_2+2\gamma\nu\mu_0\right]^2\right\}+$$
$$-\delta f(c(\sigma))c_1'(\sigma)\left\{\mu_0^c\Big[\sigma_1(\omega_1-\gamma\nu\left(1+(1-\theta)(1-2\mu_0)+\theta(1-2\mu_1)\right))^2+\right.$$
$$+(1-\sigma_1)(\omega_1-\gamma\nu\left(1+(1-\theta)(1-2\mu_0)+\theta(1-2\mu_2)\right))^2\Big]+$$
$$+(1-\mu_0^c)\Big[\sigma_2(\omega_2+\gamma\nu\left(1-(1-\theta)(1-2\mu_0)-\theta(1-2\mu_2)\right))^2+$$
$$\left.+(1-\sigma_2)(\omega_2+\gamma\nu\left(1-(1-\theta)(1-2\mu_0)-\theta(1-2\mu_1)\right))^2\Big]\right\}+$$
$$+\left[1-\delta F(c(\sigma))\right]\left\{\mu_0^c\Big[(\omega_1-\gamma\nu\left(1+(1-\theta)(1-2\mu_0)+\theta(1-2\mu_1)\right))^2+\right.$$
$$+4\sigma_1(\omega_1-\gamma\nu\left(1+(1-\theta)(1-2\mu_0)+\theta(1-2\mu_1)\right))\gamma\theta\nu\frac{\partial\mu_1}{\partial\sigma_1}+$$
$$-(\omega_1-\gamma\nu\left(1+(1-\theta)(1-2\mu_0)+\theta(1-2\mu_2)\right))^2+$$
$$+4(1-\sigma_1)(\omega_1-\gamma\nu\left(1+(1-\theta)(1-2\mu_0)+\theta(1-2\mu_2)\right))\gamma\theta\nu\frac{\partial\mu_2}{\partial\sigma_1}\Big]+$$
$$+4\gamma\theta\nu(1-\mu_0^c)\Big[\sigma_2(\omega_2+\gamma\nu\left(1-(1-\theta)(1-2\mu_0)-\theta(1-2\mu_2)\right))\frac{\partial\mu_2}{\partial\sigma_1}+$$
$$\left.+(1-\sigma_2)(\omega_2+\gamma\nu\left(1-(1-\theta)(1-2\mu_0)-\theta(1-2\mu_1)\right))\frac{\partial\mu_1}{\partial\sigma_1}\Big]\right\}=0$$

$$
\begin{aligned}
&\delta f(c(\sigma))c_2'(\sigma)\left\{\mu_0^c\left[\omega_1-2\gamma\nu(1-\mu_0)\right]^2+(1-\mu_0^c)\left[\omega_2+2\gamma\nu\mu_0\right]^2\right\}+\\
&-\delta f(c(\sigma))c_2'(\sigma)\left\{\mu_0^c\Big[\sigma_1(\omega_1-\gamma\nu\left(1+(1-\theta)(1-2\mu_0)+\theta(1-2\mu_1)\right))^2+\right.\\
&\qquad+(1-\sigma_1)(\omega_1-\gamma\nu\left(1+(1-\theta)(1-2\mu_0)+\theta(1-2\mu_2)\right))^2\Big]+\\
&\qquad+(1-\mu_0^c)\Big[\sigma_2(\omega_2+\gamma\nu\left(1-(1-\theta)(1-2\mu_0)-\theta(1-2\mu_2)\right))^2+\\
&\qquad\left.+(1-\sigma_2)(\omega_2+\gamma\nu\left(1-(1-\theta)(1-2\mu_0)-\theta(1-2\mu_1)\right))^2\Big]\right\}+\\
&+[1-\delta F(c(\sigma))]\left\{(1-\mu_0^c)\Big[(\omega_2+\gamma\nu\left(1-(1-\theta)(1-2\mu_0)-\theta(1-2\mu_2)\right))^2+\right.\\
&\qquad 4\sigma_2(\omega_2+\gamma\nu\left(1-(1-\theta)(1-2\mu_0)-\theta(1-2\mu_2)\right))\gamma\theta\nu\frac{\partial\mu_2}{\partial\sigma_2}+\\
&\qquad-(\omega_2+\gamma\nu\left(1-(1-\theta)(1-2\mu_0)-\theta(1-2\mu_1)\right))^2+\\
&\qquad 4(1-\sigma_2)(\omega_2+\gamma\nu\left(1-(1-\theta)(1-2\mu_0)-\theta(1-2\mu_1)\right))\gamma\theta\nu\frac{\partial\mu_1}{\partial\sigma_2}\Big]+\\
&\qquad+4\gamma\theta\nu\mu_0^c\Big[\sigma_1(\omega_1-\gamma\nu\left(1+(1-\theta)(1-2\mu_0)+\theta(1-2\mu_1)\right))\frac{\partial\mu_1}{\partial\sigma_2}+\\
&\qquad\left.(1-\sigma_1)(\omega_1-\gamma\nu\left(1+(1-\theta)(1-2\mu_0)+\theta(1-2\mu_2)\right))\frac{\partial\mu_2}{\partial\sigma_2}\Big]\right\}=0
\end{aligned}
$$

Applying assumptions 1-2, the F.O.C.s becomes symmetric. Thus, we consider symmetric solutions where $\sigma_1 = \sigma_2 = \sigma^*$. It follows that the solution to the central bank's information design problem solves the following condition:

$$
2\theta\gamma\nu(4\omega-2(2-\theta)\gamma\nu)\left\{[1-\delta F(c(\sigma))](2\sigma^*-1)+2\delta f(c(\sigma))c'(\sigma)\left[\sigma^*(1-\sigma^*)-\frac{1}{4}\right]\right\}=0
$$

Removing irrelevant positive terms yields:

$$
(2\omega-(2-\theta)\gamma\nu)\left\{[1-\delta F(c(\sigma))]\left(\sigma^*-\frac{1}{2}\right)+\delta f(c(\sigma))c'(\sigma)\left[\sigma^*(1-\sigma^*)-\frac{1}{4}\right]\right\}=0
$$

Observe that:

$$
\sigma^*(1-\sigma^*)-\frac{1}{4}=-\left(\sigma^*-\frac{1}{2}\right)^2
$$

and that $1-\delta F(c(\sigma)) > 0$ for any $\sigma$ and $\delta \in [0,1]$. Substituting the quadratic expression and dividing by $1 - F(c(\sigma))$ yields:

$$(2\omega - (2-\theta)\gamma\nu)\left\{\left(\sigma^* - \frac{1}{2}\right) - \frac{\delta f(c(\sigma))}{1-\delta F(c(\sigma))}c'(\sigma)\left(\sigma^* - \frac{1}{2}\right)^2\right\} = 0$$

Observe that:

$$\frac{\boldsymbol{d}\ln(1-\delta F(c(\sigma)))}{\boldsymbol{d}\sigma} = -\frac{\delta f(c(\sigma))}{1-\delta F(c(\sigma))}c'(\sigma) \leq 0$$

This implies that:

$$(2\omega - (2-\theta)\gamma\nu)\left(\sigma^* - \frac{1}{2}\right)\left[1 + \left(\frac{\boldsymbol{d}\ln(1-\delta F(c(\sigma)))}{\boldsymbol{d}\sigma}\right)\left(\sigma^* - \frac{1}{2}\right)\right] = 0$$

Accordingly, two possible candidates satisfy the necessary condition: $\underline{\sigma} = \frac{1}{2}$ and $\overline{\sigma} \in (\frac{1}{2}, 1)$ such that:

$$1 + \left(\frac{\boldsymbol{d}\ln(1-\delta F(c(\sigma)))}{\boldsymbol{d}\sigma}\right)\left(\sigma^* - \frac{1}{2}\right) = 0.$$

Therefore, $\sigma^* \in \{\underline{\sigma}, \overline{\sigma}\}$. To show which candidate is sufficient, we compare utilities across them. Central bank's utility with a generic $\widehat{\sigma}$ is:

$$\begin{aligned}
u_{\widehat{\sigma}} = &- \delta F(c(\sigma))(\omega - \gamma\nu)^2 \\
&- (1-\delta F(c(\sigma)))\left[\widehat{\sigma}(\omega - \gamma\nu(1+\theta(1-2\widehat{\sigma})))^2 + (1-\widehat{\sigma})(\omega - \gamma\nu(1-\theta(1-2\widehat{\sigma})))^2\right]
\end{aligned}$$

Its utility with uninformative communication $\underline{\sigma}$ is:

$$u_{\underline{\sigma}} = -(\omega - \gamma\nu)^2$$

For $\sigma^* = \underline{\sigma}$ to be optimal, $u_{\underline{\sigma}} - u_{\widehat{\sigma}} \geq 0$. Under which condition does this relationship hold?

In the first step, $u_{\widehat{\sigma}}$ can be rewritten into:

$$\begin{aligned}
u_{\widehat{\sigma}} = &- \delta F(c(\sigma))(\omega - \gamma\nu)^2 \\
&- (1-\delta F(c(\sigma)))(\omega - \gamma\nu)^2 + (1-\delta F(c(\sigma)))(\omega - \gamma\nu)^2 \\
&- (1-\delta F(c(\sigma)))\left[\widehat{\sigma}(\omega - \gamma\nu(1+\theta(1-2\widehat{\sigma})))^2 + (1-\widehat{\sigma})(\omega - \gamma\nu(1-\theta(1-2\widehat{\sigma})))^2\right]
\end{aligned}$$

It follows that:

$$u_{\widehat{\sigma}} = u_{\underline{\sigma}} - (1 - \delta F(c(\sigma)))\Big[\widehat{\sigma}(\omega - \gamma\nu(1 + \theta(1 - 2\widehat{\sigma})))^2$$
$$+(1 - \widehat{\sigma})(\omega - \gamma\nu(1 - \theta(1 - 2\widehat{\sigma})))^2 - (\omega - \gamma\nu)^2\Big]$$

Thus, showing $u_{\underline{\sigma}} - u_{\widehat{\sigma}} \geq 0$ is equivalent to show:

$$\widehat{\sigma}(\omega - \gamma\nu(1 + \theta(1 - 2\widehat{\sigma})))^2 + (1 - \widehat{\sigma})(\omega - \gamma\nu(1 - \theta(1 - 2\widehat{\sigma})))^2 - (\omega - \gamma\nu)^2 \geq 0$$

In the second step,

$$\begin{aligned}
\widehat{\sigma}(\omega - \gamma\nu(1 + \theta(1 - 2\widehat{\sigma})))^2 &= \widehat{\sigma}[\omega^2 - 2\omega\gamma\nu(1 + \theta(1 - 2\widehat{\sigma})) + \gamma^2\nu^2(1 + \theta(1 - 2\widehat{\sigma}))^2] \\
(1 - \widehat{\sigma})(\omega - \gamma\nu(1 - \theta(1 - 2\widehat{\sigma})))^2 &= (1 - \widehat{\sigma})[\omega^2 - 2\omega\gamma\nu(1 - \theta(1 - 2\widehat{\sigma})) + \gamma^2\nu^2(1 - \theta(1 - 2\widehat{\sigma}))^2] \\
(\omega - \gamma\nu)^2 &= \omega^2 - 2\omega\gamma\nu + \gamma^2\nu^2
\end{aligned}$$

The above inequality can be rearranged as:

$$-2\omega\gamma\nu[1 - \theta(1 - 2\widehat{\sigma})^2] + \gamma^2\nu^2[1 + \theta^2(1 - 2\widehat{\sigma})^2 - 2\theta(1 - 2\widehat{\sigma})^2] + 2\omega\gamma\nu - \gamma^2\nu^2 \geq 0$$

It can be further expressed nicely as:

$$\begin{aligned}
&2\omega\gamma\nu\theta(1 - 2\widehat{\sigma})^2 + \gamma^2\nu^2\theta(\theta - 2)(1 - 2\widehat{\sigma})^2 \geq 0 \\
&\gamma\nu\theta(1 - 2\widehat{\sigma})^2[2\omega - \gamma\nu(2 - \theta)] \geq 0
\end{aligned}$$

Given that $\gamma\nu\theta \geq 0$ by construction and $(1 - 2\widehat{\sigma})^2 \geq 0$ for any $\widehat{\sigma} \in [\frac{1}{2}, 1)$, we find that, consistent with the necessary condition, $\sigma^* = \underline{\sigma} = \frac{1}{2}$ if $2\omega - \gamma\nu(2 - \theta) \geq 0$ while $\sigma^* = \overline{\sigma} \in (\frac{1}{2}, 1)$ if $2\omega - \gamma\nu(2 - \theta) < 0$.

# B Additional Figures for Robustness Checks

Figure 7: Comparative statics for $\mu_0$ and $\mu_0^c$ with $\delta = 0.5$, holding fixed all other parameters as in the benchmark.

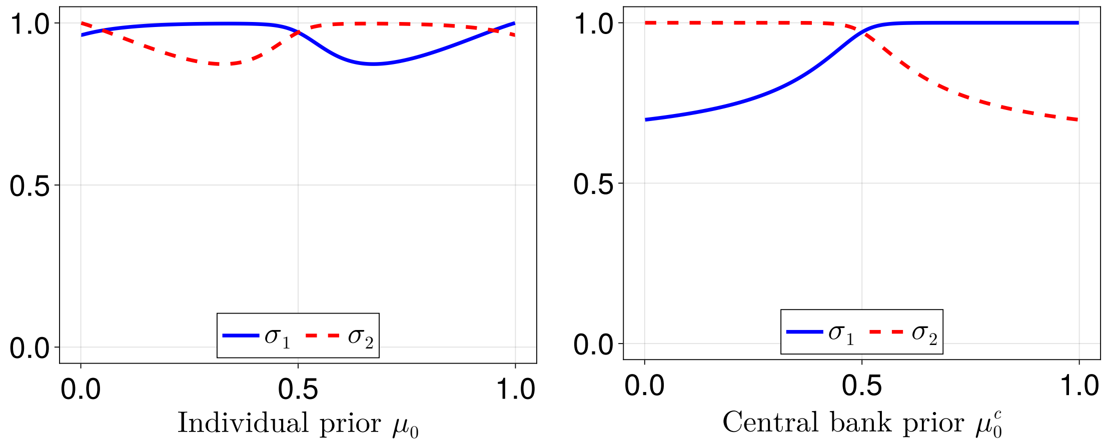


Figure 8: Comparative statics for $\mu_0^c$ with $\gamma = 1$, holding fixed all other parameters as in the benchmark.

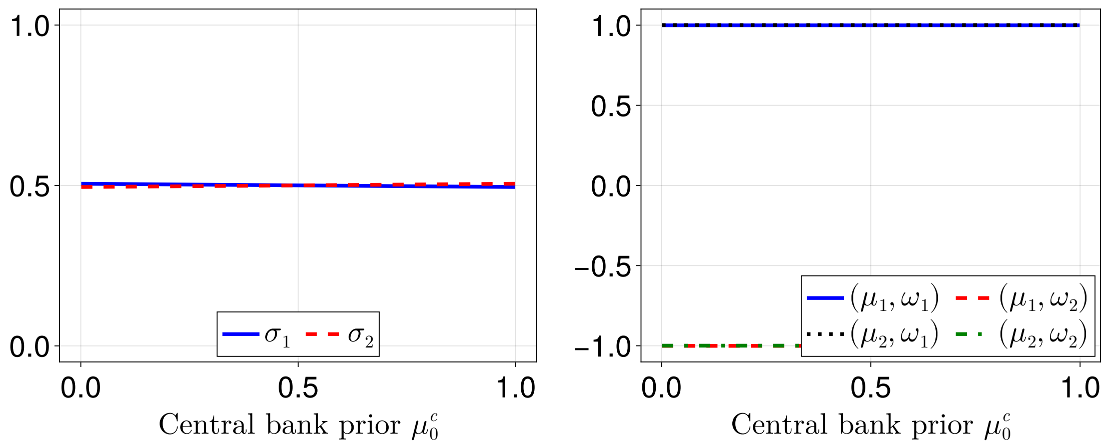

Figure 9: Comparative statics for $\mu_0$ with $\gamma = 1$ and $\delta = 0.5$, holding fixed all other parameters as in the benchmark.

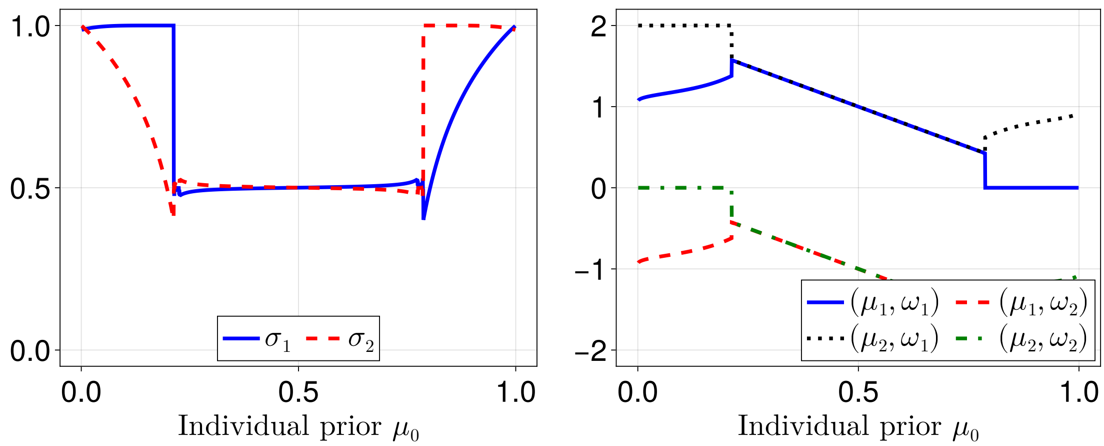


Figure 10: Comparative statics for $\mu_0^c$ with $\mu_0 = 0.1$, $\gamma = 1$, and $\delta = 0.5$, holding fixed all other parameters as in the benchmark.

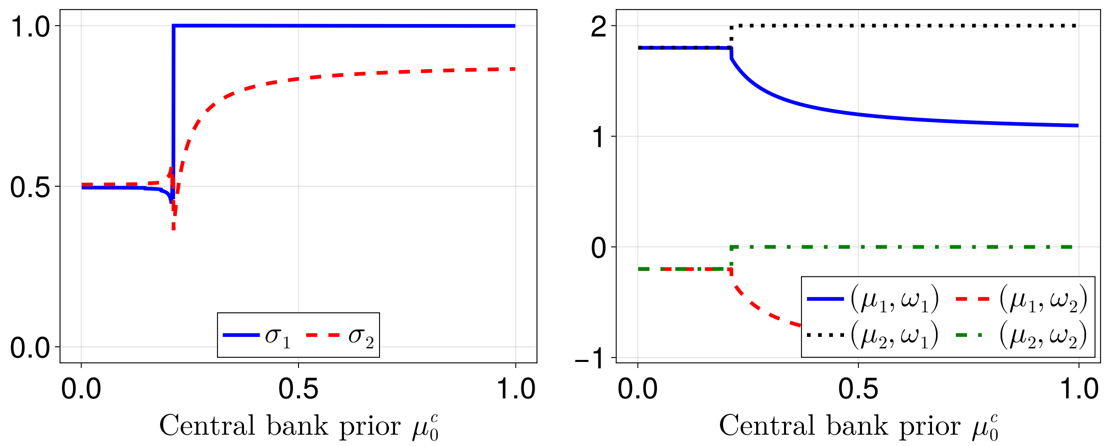

Figure 11: Comparative statics for $\mu_0$ with $(\omega_1, \omega_2) = (2, -2)$, holding fixed all other parameters as in the benchmark.

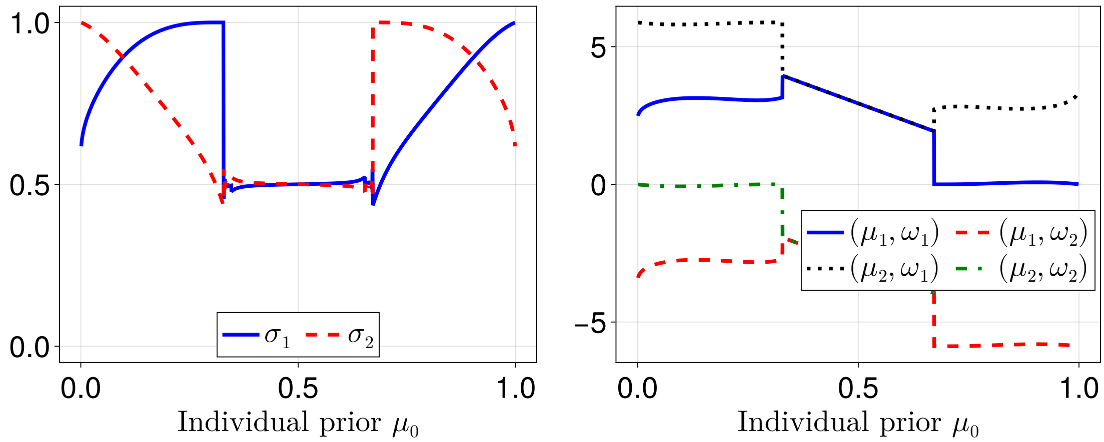


Figure 12: Comparative statics for $\mu_0^c$ with $(\omega_1, \omega_2) = (2, -2)$, holding fixed all other parameters as in the benchmark.

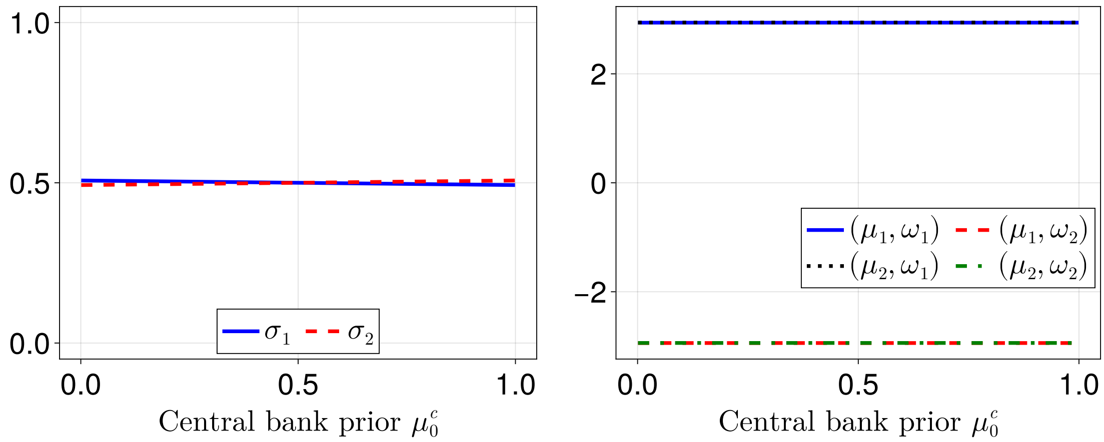

Figure 13: Comparative statics for $\mu_0$ with $(\omega_1, \omega_2) = (2, -1)$, holding fixed all other parameters as in the benchmark.

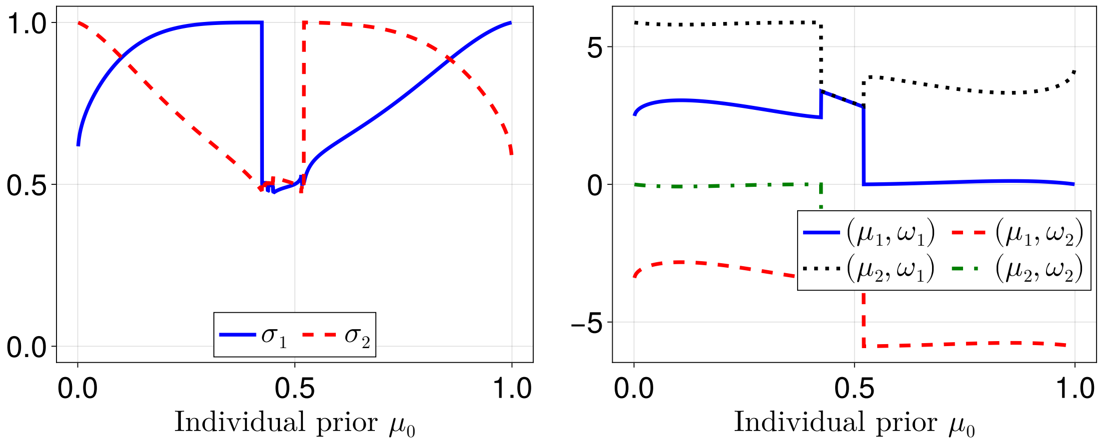


Figure 14: Comparative statics for $\mu_0^c$ with $(\omega_1, \omega_2) = (2, -1)$, holding fixed all other parameters as in the benchmark.

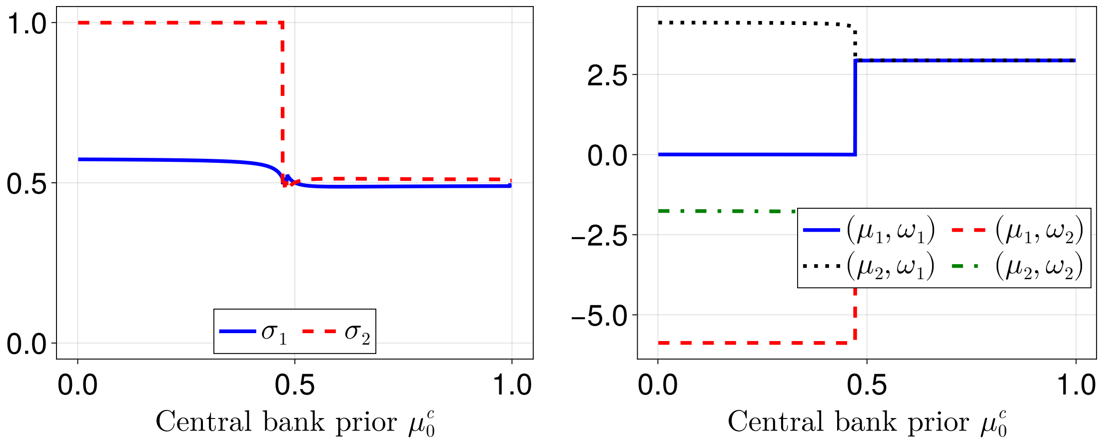

Figure 15: Comparative statics for $\mu_0$ with $\gamma = 1$ and $(\omega_1, \omega_2) = (2, -2)$, holding fixed all other parameters as in the benchmark.

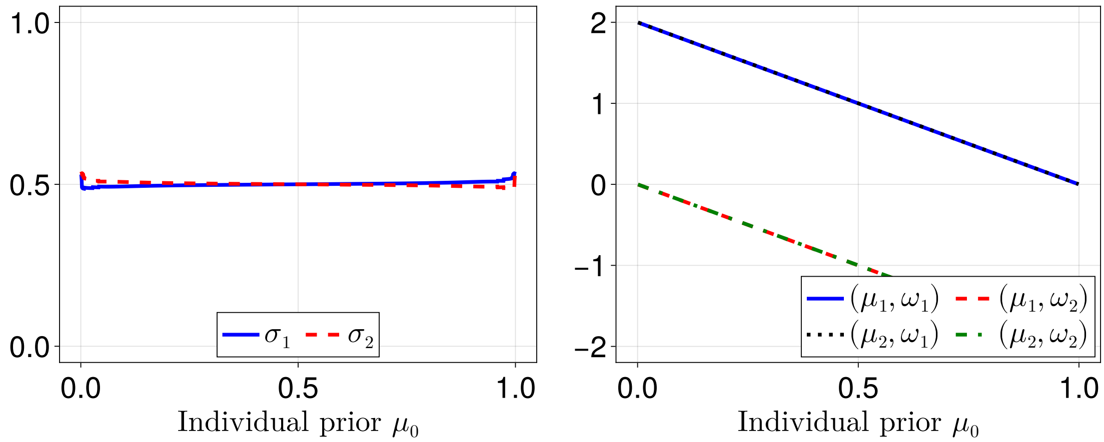


Figure 16: Comparative statics for $\mu_0^c$ with $\mu_0 = 0.1$, $\gamma = 1$, and $(\omega_1, \omega_2) = (2, -2)$, holding fixed all other parameters as in the benchmark.

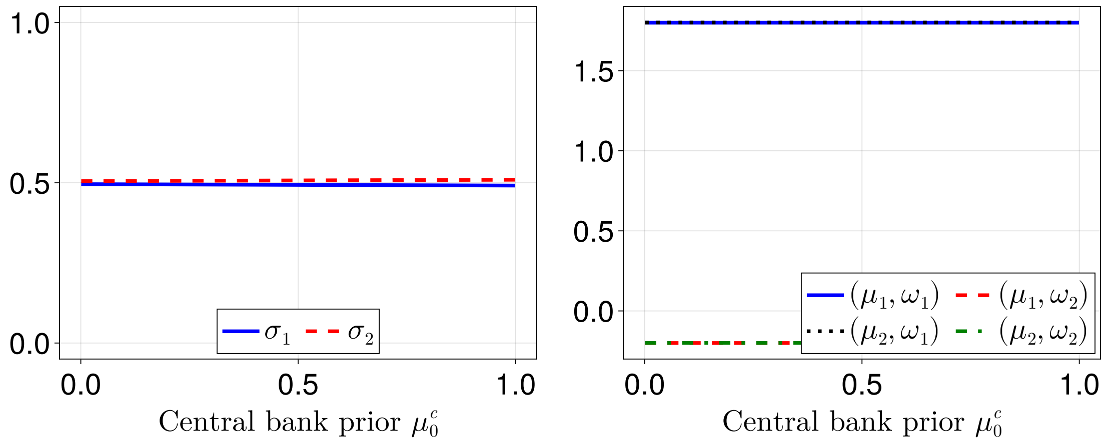

Figure 17: Comparative statics for $\mu_0$ with $\gamma = 1$ and $(\omega_1, \omega_2) = (2, -1)$, holding fixed all other parameters as in the benchmark.

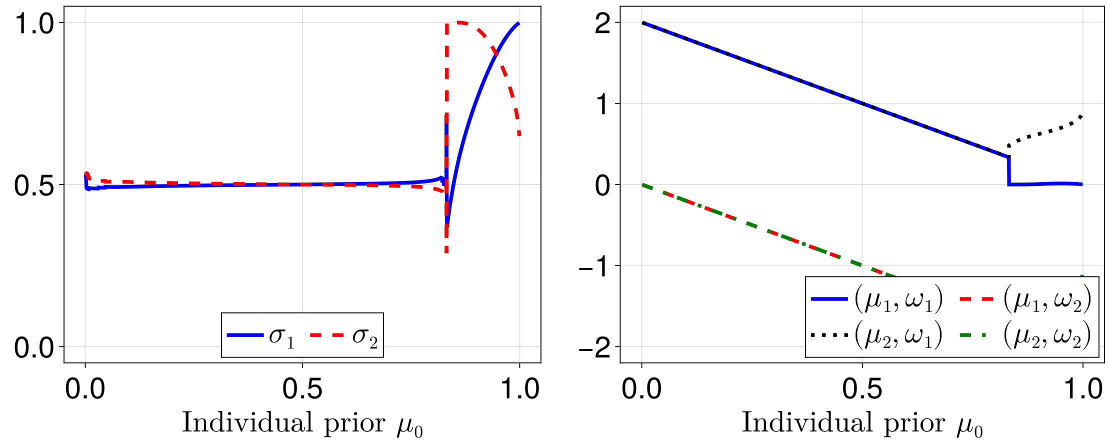


Figure 18: Comparative statics for $\mu_0$ with $\gamma = 1$ and $\delta = 1$, for right-skewed (on the left) and left-skewed (on the right) distributions of the attention budget.

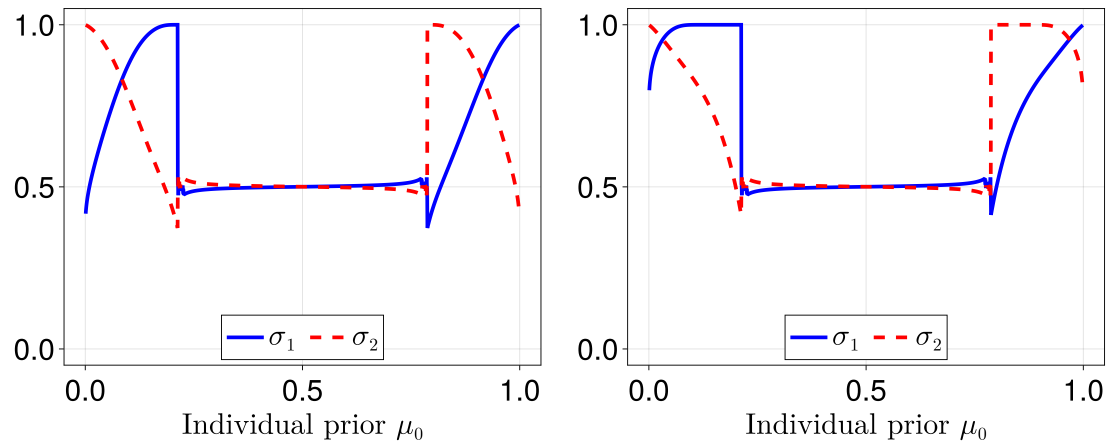

Figure 19: Comparative statics for $\mu_0$ with $\gamma = 1$ and $\delta = 0.5$, for right-skewed (on the left) and left-skewed (on the right) distributions of the attention budget.